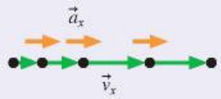
2 Kinematics in One Dimension

This Japanese "bullet train" accelerates slowly but steadily until reaching a speed of 300 km/h.

IN THIS CHAPTER, you will learn to solve problems about motion along a straight line.

What is kinematics?

Kinematics is the mathematical description of motion. We begin with motion along a straight line. Our primary tools will be an object's position, velocity, and acceleration.

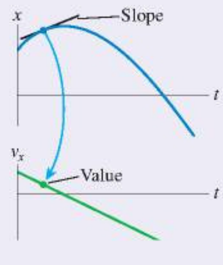


« LOOKING BACK Sections 1.4–1.6 Velocity, acceleration, and Tactics Box 1.4 about signs

How are graphs used in kinematics?

Graphs are a very important visual representation of motion, and learning to "think graphically" is one of our goals. We'll work with graphs showing how position, velocity, and acceleration change with time. These graphs are related to each other:

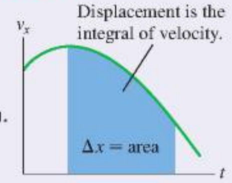
- Velocity is the slope of the position graph.
- Acceleration is the slope of the velocity graph.



How is calculus used in kinematics?

Motion is change, and calculus is the mathematical tool for describing a quantity's rate of change. We'll find that

- Velocity is the time derivative of position.
- Acceleration is the time derivative of velocity.



What are models?

A model is a simplified description of a situation that focuses on essential features while ignoring many details. Models allow us to make sense of complex situations by seeing them as variations on a common theme, all with the same underlying physics.

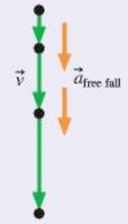
MODEL 2.1

Look for model boxes like this throughout the book.

- Key figures
- Key equations
- Model limitations

What is free fall?

Free fall is motion under the influence of gravity only. Free fall is not literally "falling" because it also applies to objects thrown straight up and to projectiles. Surprisingly, all objects in free fall, *regardless of their mass*, have the same acceleration. Motion on a frictionless inclined plane is closely related to free-fall motion.



How will I use kinematics?

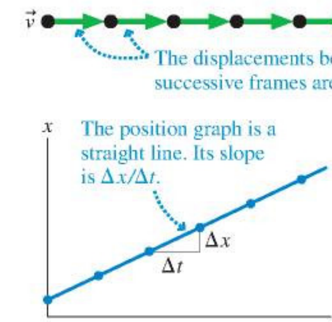
The equations of motion that you learn in this chapter will be used throughout the entire book. In Part I, we'll see how an object's motion is related to forces acting on the object. We'll later apply these kinematic equations to the motion of waves and to the motion of charged particles in electric and magnetic fields.

2.1 Uniform Motion

The simplest possible motion is motion along a straight line at a constant, unvarying speed. We call this **uniform motion.** Because velocity is the combination of speed and direction, **uniform motion is motion with constant velocity.**

FIGURE 2.1 shows the motion diagram of an object in uniform motion. For example, this might be you riding your bicycle along a straight line at a perfectly steady 5 m/s ($\approx$ 10 mph). Notice how all the displacements are exactly the same; this is a characteristic of uniform motion.

FIGURE 2.1 Motion diagram and position graph for uniform motion.



If we make a position-versus-time graph—remember that position is graphed on the *vertical* axis—it's a straight line. In fact, an alternative definition is that **an object's motion is uniform if and only if its position-versus-time graph is a straight line.**

« Section 1.4 defined an object's **average velocity** as $\Delta\vec{r}/\Delta t$. For one-dimensional motion, this is simply $\Delta x/\Delta t$ (for horizontal motion) or $\Delta y/\Delta t$ (for vertical motion). You can see in Figure 2.1 that Δx and Δt are, respectively, the "rise" and "run" of the position graph. Because rise over run is the slope of a line,

$$v_{avg} \equiv \frac{\Delta x}{\Delta t} \quad \text{or} \quad \frac{\Delta y}{\Delta t} = \text{slope of the position-versus-time graph} \qquad (2.1)$$

That is, **the average velocity is the slope of the position-versus-time graph.** Velocity has units of "length per time," such as "miles per hour." The SI units of velocity are meters per second, abbreviated m/s.

NOTE The symbol $\equiv$ in Equation 2.1 stands for "is defined as." This is a stronger statement than the two sides simply being equal.

The constant slope of a straight-line graph is another way to see that the velocity is constant for uniform motion. There's no real need to specify "average" for a velocity that doesn't change, so we will drop the subscript and refer to the average velocity as v_x or v_y.

An object's **speed** v is how fast it's going, independent of direction. This is simply $v = |v_x|$ or $v = |v_y|$, the magnitude or absolute value of the object's velocity. Although we will use speed from time to time, our mathematical analysis of motion is based on velocity, not speed. The subscript in v_x or v_y is an essential part of the notation, reminding us that, even in one dimension, the velocity is a vector.

EXAMPLE 2.1 Relating a velocity graph to a position graph

FIGURE 2.2 is the position-versus-time graph of a car.

a. Draw the car's velocity-versus-time graph.

b. Describe the car's motion.

MODEL Model the car as a particle, with a well-defined position at each instant of time.

VISUALIZE Figure 2.2 is the graphical representation.

SOLVE a. The car's position-versus-time graph is a sequence of three straight lines. Each of these straight lines represents uniform motion at a constant velocity. We can determine the car's velocity during each interval of time by measuring the slope of the line.

The position graph starts out sloping downward—a negative slope. Although the car moves a distance of 4.0 m during the first 2.0 s, its *displacement* is

$$\Delta x = x_{\text{at } 2.0\text{ s}} - x_{\text{at } 0.0\text{ s}} = -4.0\text{ m} - 0.0\text{ m} = -4.0\text{ m}$$

The time interval for this displacement is $\Delta t = 2.0$ s, so the velocity during this interval is

$$v_x = \frac{\Delta x}{\Delta t} = \frac{-4.0\text{ m}}{2.0\text{ s}} = -2.0\text{ m/s}$$

The car's position does not change from $t = 2$ s to $t = 4$ s ($\Delta x = 0$), so $v_x = 0$. Finally, the displacement between $t = 4$ s and $t = 6$ s is $\Delta x = 10.0$ m. Thus the velocity during this interval is

$$v_x = \frac{10.0\text{ m}}{2.0\text{ s}} = 5.0\text{ m/s}$$

These velocities are shown on the velocity-versus-time graph of FIGURE 2.3.

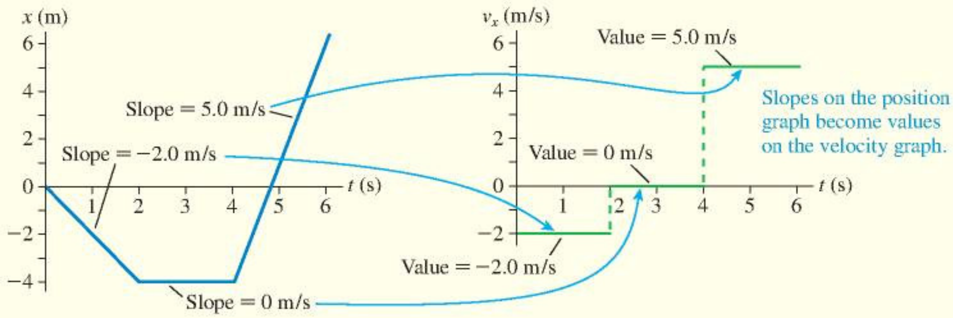
b. The car backs up for 2 s at 2.0 m/s, sits at rest for 2 s, then drives forward at 5.0 m/s for at least 2 s. We can't tell from the graph what happens for $t > 6$ s.

ASSESS The velocity graph and the position graph look completely different. The *value* of the velocity graph at any instant of time equals the *slope* of the position graph.

Continued

FIGURE 2.2 Position-versus-time graph.

FIGURE 2.3 The corresponding velocity-versus-time graph.



Example 2.1 brought out several points that are worth emphasizing.

TACTICS BOX 2.1

Interpreting position-versus-time graphs

1. Steeper slopes correspond to faster speeds.
2. Negative slopes correspond to negative velocities and, hence, to motion to the left (or down).
3. The slope is a ratio of intervals, $\Delta x/\Delta t$, not a ratio of coordinates. That is, the slope is *not* simply x/t.

Exercises 1–3

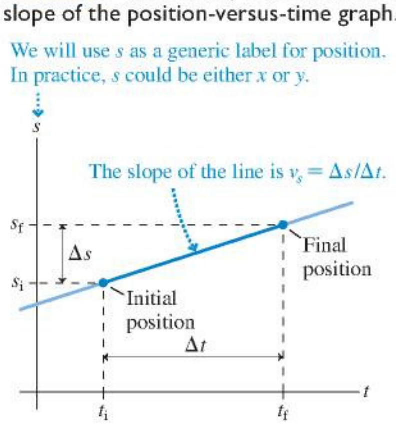
NOTE We are distinguishing between the *actual* slope and the *physically meaningful* slope. If you were to use a ruler to measure the rise and the run of the graph, you could compute the actual slope of the line as drawn on the page. That is not the slope to which we are referring when we equate the velocity with the slope of the line. Instead, we find the *physically meaningful* slope by measuring the rise and run using the scales along the axes. The "rise" Δx is some number of meters; the "run" Δt is some number of seconds. The physically meaningful rise and run include units, and the ratio of these units gives the units of the slope.

The Mathematics of Uniform Motion

The physics of the motion is the same regardless of whether an object moves along the x-axis, the y-axis, or any other straight line. Consequently, it will be convenient to write equations for a "generic axis" that we will call the s-axis. The position of an object will be represented by the symbol s and its velocity by v_s.

NOTE In a specific problem you should use either x or y rather than s.

FIGURE 2.4 The velocity is found from the slope of the position-versus-time graph.



Consider an object in uniform motion along the s-axis with the linear position-versus-time graph shown in FIGURE 2.4. The object's **initial position** is s_i at time t_i. The term *initial position* refers to the starting point of our analysis or the starting point in a problem; the object may or may not have been in motion prior to t_i. At a later time t_f, the ending point of our analysis, the object's **final position** is s_f.

The object's velocity v_s along the s-axis can be determined by finding the slope of the graph:

$$v_s = \frac{\text{rise}}{\text{run}} = \frac{\Delta s}{\Delta t} = \frac{s_f - s_i}{t_f - t_i} \tag{2.2}$$

Equation 2.2 is easily rearranged to give

$$s_f = s_i + v_s\,\Delta t \qquad \text{(uniform motion)} \qquad (2.3)$$

Equation 2.3 tells us that the object's position increases linearly as the elapsed time Δt increases—exactly as we see in the straight-line position graph.

The Uniform-Motion Model

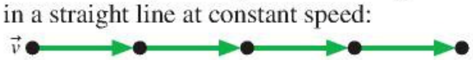
Chapter 1 introduced a *model* as a simplified picture of reality, but one that still captures the essence of what we want to study. When it comes to motion, few real objects move with a precisely constant velocity. Even so, there are many cases in which it is quite reasonable to model their motion as being uniform. That is, uniform motion is a very good approximation of their actual, but more complex, motion. The **uniform-motion model** is a coherent set of representations—words, pictures, graphs, and equations—that allows us to explain an object's motion and to predict where the object will be at a future instant of time.

MODEL 2.1

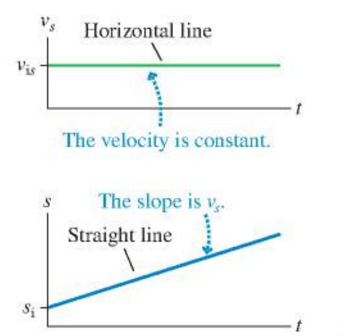
Uniform motion

For motion with constant velocity.

- Model the object as a particle moving in a straight line at constant speed:



- Mathematically:
  - $v_s = \Delta s/\Delta t$
  - $s_f = s_i + v_s\,\Delta t$
- Limitations: Model fails if the particle has a significant change of speed or direction.



Exercise 4

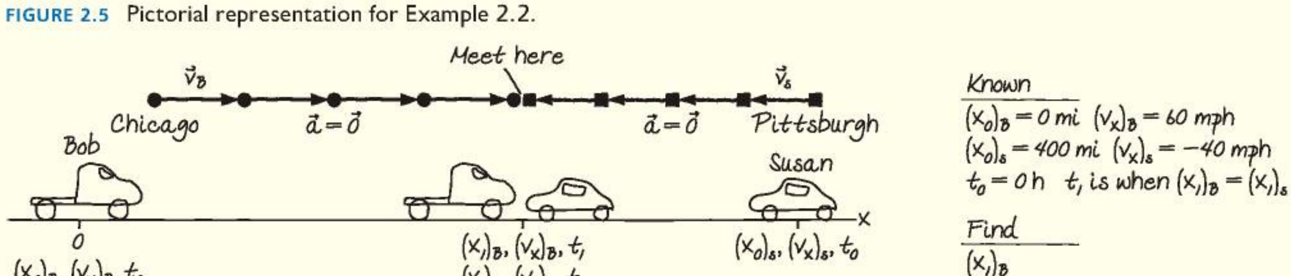
EXAMPLE 2.2 Lunch in Cleveland?

Bob leaves home in Chicago at 9:00 A.M. and drives east at 60 mph. Susan, 400 miles to the east in Pittsburgh, leaves at the same time and travels west at 40 mph. Where will they meet for lunch?

MODEL Here is a problem where, for the first time, we can really put all four aspects of our problem-solving strategy into play. To begin, we'll model Bob's and Susan's cars as being in uniform motion. Their real motion is certainly more complex, but over a long drive it's reasonable to approximate their motion as constant speed along a straight line.

VISUALIZE FIGURE 2.5 shows the pictorial representation. The equal spacings of the dots in the motion diagram indicate that the motion is uniform. In evaluating the given information, we

FIGURE 2.5 Pictorial representation for Example 2.2.



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recognize that the starting time of 9:00 A.M. is not relevant to the problem. Consequently, the initial time is chosen as simply $t_0 = 0$ h. Bob and Susan are traveling in opposite directions, hence one of the velocities must be a negative number. We have chosen a coordinate system in which Bob starts at the origin and moves to the right (east) while Susan is moving to the left (west). Thus Susan has the negative velocity. Notice how we've assigned position, velocity, and time symbols to each point in the motion. Pay special attention to how subscripts are used to distinguish different points in the problem and to distinguish Bob's symbols from Susan's.

One purpose of the pictorial representation is to establish what we need to find. Bob and Susan meet when they have the same position at the same time t_1. Thus we want to find $(x_1)_B$ at the time when $(x_1)_B = (x_1)_S$. Notice that $(x_1)_B$ and $(x_1)_S$ are Bob's and Susan's *positions,* which are equal when they meet, not the distances they have traveled.

SOLVE The goal of the mathematical representation is to proceed from the pictorial representation to a mathematical solution of the problem. We can begin by using Equation 2.3 to find Bob's and Susan's positions at time t_1 when they meet:

$$(x_1)_B = (x_0)_B + (v_x)_B(t_1 - t_0) = (v_x)_B t_1$$
$$(x_1)_S = (x_0)_S + (v_x)_S(t_1 - t_0) = (x_0)_S + (v_x)_S t_1$$

Notice two things. First, we started by writing the *full* statement of Equation 2.3. Only then did we simplify by dropping those terms known to be zero. You're less likely to make accidental errors if you follow this procedure. Second, we replaced the generic symbol s with the specific horizontal-position symbol x, and we replaced the generic subscripts i and f with the specific symbols 0 and 1 that we defined in the pictorial representation. This is also good problem-solving technique.

The condition that Bob and Susan meet is

$$(x_1)_B = (x_1)_S$$

By equating the right-hand sides of the above equations, we get

$$(v_x)_B t_1 = (x_0)_S + (v_x)_S t_1$$

Solving for t_1 we find that they meet at time

$$t_1 = \frac{(x_0)_S}{(v_x)_B - (v_x)_S} = \frac{400 \text{ miles}}{60 \text{ mph} - (-40) \text{ mph}} = 4.0 \text{ hours}$$

Finally, inserting this time back into the equation for $(x_1)_B$ gives

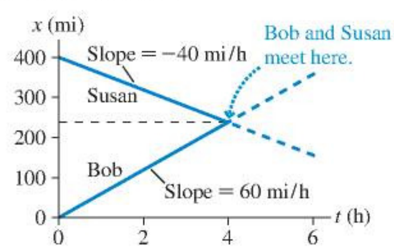
$$(x_1)_B = \left(60 \frac{\text{miles}}{\text{hour}}\right) \times (4.0 \text{ hours}) = 240 \text{ miles}$$

As noted in Chapter 1, this textbook will assume that all data are good to at least two significant figures, even when one of those is a trailing zero. So 400 miles, 60 mph, and 40 mph each have two significant figures, and consequently we've calculated results to two significant figures.

While 240 miles is a number, it is not yet the answer to the question. The phrase "240 miles" by itself does not say anything meaningful. Because this is the value of Bob's *position,* and Bob was driving east, the answer to the question is, "They meet 240 miles east of Chicago."

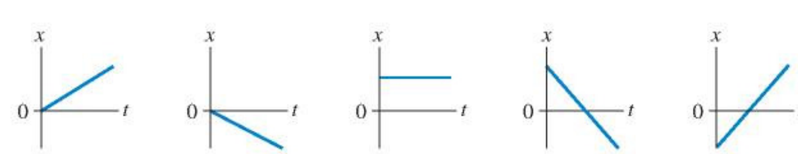
ASSESS Before stopping, we should check whether or not this answer seems reasonable. We certainly expected an answer between 0 miles and 400 miles. We also know that Bob is driving faster than Susan, so we expect that their meeting point will be *more* than halfway from Chicago to Pittsburgh. Our assessment tells us that 240 miles is a reasonable answer.

FIGURE 2.6 Position-versus-time graphs for Bob and Susan.



It is instructive to look at this example from a graphical perspective. **FIGURE 2.6** shows position-versus-time graphs for Bob and Susan. Notice the negative slope for Susan's graph, indicating her negative velocity. The point of interest is the intersection of the two lines; this is where Bob and Susan have the same position at the same time. Our method of solution, in which we equated $(x_1)_B$ and $(x_1)_S$, is really just solving the mathematical problem of finding the intersection of two lines. This procedure is useful for many problems in which there are two moving objects.

STOP TO THINK 2.1 Which position-versus-time graph represents the motion shown in the motion diagram?



(a) (b) (c) (d) (e)

2.2 Instantaneous Velocity

Uniform motion is simple, but objects rarely travel for long with a constant velocity. Far more common is a velocity that changes with time. For example, **FIGURE 2.7** shows the motion diagram and position graph of a car speeding up after the light turns green. Notice how the velocity vectors increase in length, causing the graph to curve upward as the car's displacements get larger and larger.

If you were to watch the car's speedometer, you would see it increase from 0 mph to 10 mph to 20 mph and so on. At any instant of time, the speedometer tells you how fast the car is going *at that instant*. If we include directional information, we can define an object's **instantaneous velocity**—speed and direction—as its velocity at a single instant of time.

For uniform motion, the slope of the straight-line position graph is the object's velocity. **FIGURE 2.8** shows that there's a similar connection between instantaneous velocity and the slope of a curved position graph.

FIGURE 2.7 Motion diagram and position graph of a car speeding up.

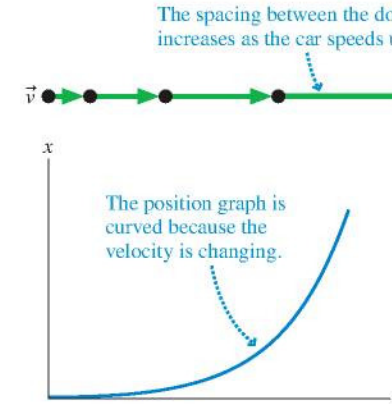
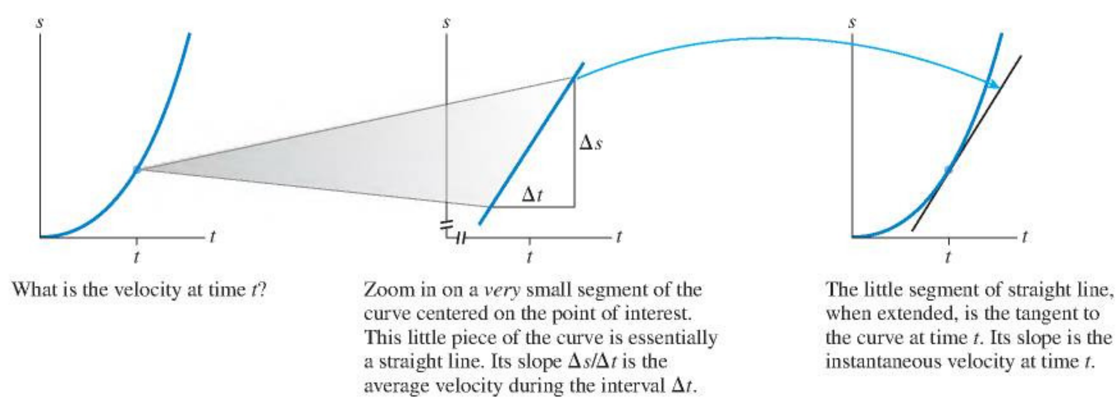


FIGURE 2.8 Instantaneous velocity at time t is the slope of the tangent to the curve at that instant.



What we see graphically is that the average velocity $v_{avg} = \Delta s/\Delta t$ becomes a better and better approximation to the instantaneous velocity v_s as the time interval Δt over which the average is taken gets smaller and smaller. We can state this idea mathematically in terms of the limit $\Delta t \rightarrow 0$:

$$v_s \equiv \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \quad \text{(instantaneous velocity)} \tag{2.4}$$

As Δt continues to get smaller, the average velocity $v_{avg} = \Delta s/\Delta t$ reaches a constant or *limiting* value. That is, **the instantaneous velocity at time t is the average velocity during a time interval Δt, centered on t, as Δt approaches zero.** In calculus, this limit is called *the derivative of s with respect to t*, and it is denoted ds/dt.

Graphically, $\Delta s/\Delta t$ is the slope of a straight line. As Δt gets smaller (i.e., more and more magnification), the straight line becomes a better and better approximation of the curve *at that one point*. In the limit $\Delta t \rightarrow 0$, the straight line is tangent to the curve. As Figure 2.8 shows, **the instantaneous velocity at time t is the slope of the line that is tangent to the position-versus-time graph at time t.** That is,

$$v_s = \text{slope of the position-versus-time graph at time } t \tag{2.5}$$

The steeper the slope, the larger the magnitude of the velocity.

EXAMPLE 2.3 Finding velocity from position graphically

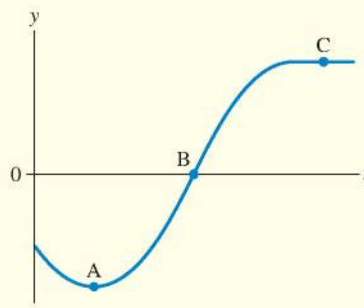
FIGURE 2.9 shows the position-versus-time graph of an elevator.

a. At which labeled point or points does the elevator have the least velocity?

b. At which point or points does the elevator have maximum velocity?

c. Sketch an approximate velocity-versus-time graph for the elevator.

FIGURE 2.9 Position-versus-time graph.



MODEL Model the elevator as a particle.

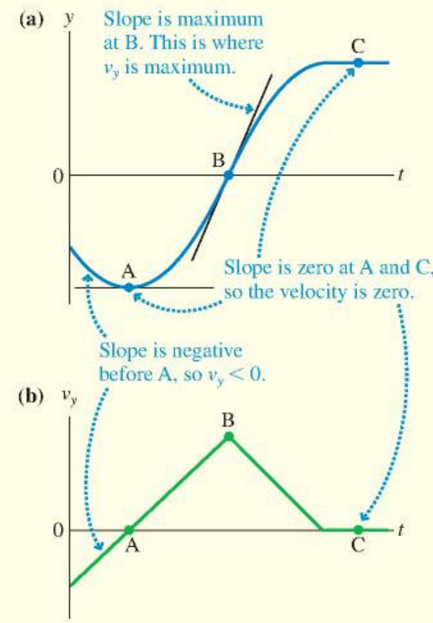
VISUALIZE Figure 2.9 is the graphical representation.

SOLVE a. At any instant, an object's velocity is the slope of its position graph. FIGURE 2.10a shows that the elevator has the least velocity—no velocity at all!—at points A and C where the slope is zero. At point A, the velocity is only instantaneously zero. At point C, the elevator has actually stopped and remains at rest.

b. The elevator has maximum velocity at B, the point of steepest slope.

c. Although we cannot find an exact velocity-versus-time graph, we can see that the slope, and hence v_y, is initially negative, becomes zero at point A, rises to a maximum value at point B, decreases back to zero a little before point C, then remains at zero thereafter.

FIGURE 2.10 The velocity-versus-time graph is found from the slope of the position graph.



Thus FIGURE 2.10b shows, at least approximately, the elevator's velocity-versus-time graph.

ASSESS Once again, the shape of the velocity graph bears no resemblance to the shape of the position graph. You must transfer *slope* information from the position graph to *value* information on the velocity graph.

Scientists and engineers must use calculus to calculate the orbits of satellites.

A Little Calculus: Derivatives

Calculus—invented simultaneously in England by Newton and in Germany by Leibniz—is designed to deal with instantaneous quantities. In other words, it provides us with the tools for evaluating limits such as the one in Equation 2.4.

The notation ds/dt is called *the derivative of s with respect to t*, and Equation 2.4 defines it as the limiting value of a ratio. As Figure 2.8 showed, ds/dt can be interpreted graphically as the slope of the line that is tangent to the position graph.

The most common functions we will use in Parts I and II of this book are powers and polynomials. Consider the function $u(t) = ct^n$, where c and n are constants. The symbol u is a "dummy name" to represent any function of time, such as $x(t)$ or $y(t)$. The following result is proven in calculus:

$$\text{The derivative of } u = ct^n \text{ is } \frac{du}{dt} = nct^{n-1} \tag{2.6}$$

For example, suppose the position of a particle as a function of time is $s(t) = 2t^2$ m, where t is in s. We can find the particle's velocity $v_s = ds/dt$ by using Equation 2.6 with $c = 2$ and $n = 2$ to calculate

$$v_s = \frac{ds}{dt} = 2 \cdot 2t^{2-1} = 4t$$

This is an expression for the particle's velocity as a function of time.

FIGURE 2.11 shows the particle's position and velocity graphs. It is critically important to understand the relationship between these two graphs. The *value* of the velocity graph at any instant of time, which we can read directly off the vertical axis, is the *slope* of the position graph at that same time. This is illustrated at $t = 3$ s.

A value that doesn't change with time, such as the position of an object at rest, can be represented by the function $u = c =$ constant. That is, the exponent of t^n is $n = 0$. You can see from Equation 2.6 that the derivative of a constant is zero. That is,

$$\frac{du}{dt} = 0 \text{ if } u = c = \text{constant} \tag{2.7}$$

This makes sense. The graph of the function $u = c$ is simply a horizontal line. The slope of a horizontal line—which is what the derivative du/dt measures—is zero.

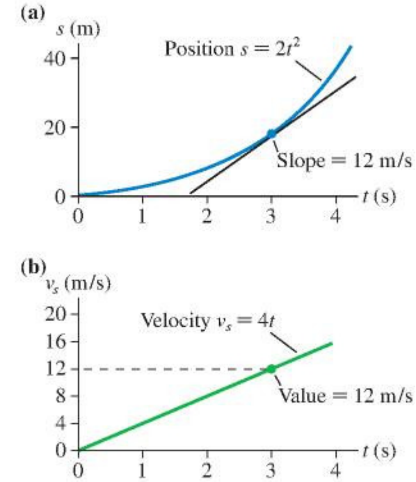
The only other information we need about derivatives for now is how to evaluate the derivative of the sum of two functions. Let u and w be two separate functions of time. You will learn in calculus that

$$\frac{d}{dt}(u + w) = \frac{du}{dt} + \frac{dw}{dt} \tag{2.8}$$

That is, the derivative of a sum is the sum of the derivatives.

NOTE You may have learned in calculus to take the derivative dy/dx, where y is a function of x. The derivatives we use in physics are the same; only the notation is different. We're interested in how quantities change with time, so our derivatives are with respect to t instead of x.

FIGURE 2.11 Position-versus-time graph and the corresponding velocity-versus-time graph.



EXAMPLE 2.4 Using calculus to find the velocity

A particle's position is given by the function $x(t) = (-t^3 + 3t)$ m, where t is in s.

a. What are the particle's position and velocity at $t = 2$ s?

b. Draw graphs of x and v_x during the interval $-3 \text{ s} \le t \le 3$ s.

c. Draw a motion diagram to illustrate this motion.

SOLVE

a. We can compute the position directly from the function x:

$$x(\text{at } t = 2 \text{ s}) = -(2)^3 + (3)(2) = -8 + 6 = -2 \text{ m}$$

The velocity is $v_x = dx/dt$. The function for x is the sum of two polynomials, so

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-t^3 + 3t) = \frac{d}{dt}(-t^3) + \frac{d}{dt}(3t)$$

The first derivative is a power with $c = -1$ and $n = 3$; the second has $c = 3$ and $n = 1$. Using Equation 2.6, we have

$$v_x = (-3t^2 + 3) \text{ m/s}$$

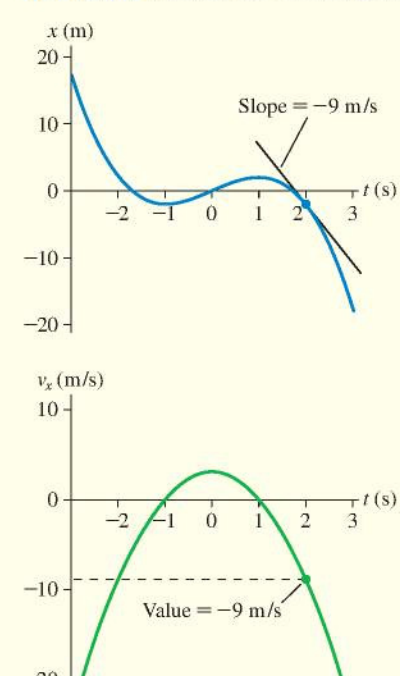
where t is in s. Evaluating the velocity at $t = 2$ s gives

$$v_x(\text{at } t = 2 \text{ s}) = -3(2)^2 + 3 = -9 \text{ m/s}$$

The negative sign indicates that the particle, at this instant of time, is moving to the *left* at a speed of 9 m/s.

b. **FIGURE 2.12** shows the position graph and the velocity graph. You can make graphs like these with a graphing calculator or graphing software. The slope of the position-versus-time graph at $t = 2$ s is -9 m/s; this becomes the *value* that is graphed for the velocity at $t = 2$ s.

FIGURE 2.12 Position and velocity graphs.



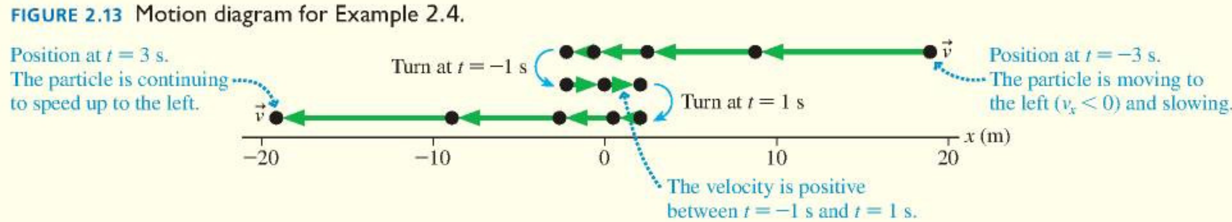
Continued

c. Finally, we can interpret the graphs in Figure 2.12 to draw the motion diagram shown in **FIGURE 2.13**.

- The particle is initially to the right of the origin ($x > 0$ at $t = -3$ s) but moving to the left ($v_x < 0$). Its *speed* is slowing ($v = |v_x|$ is decreasing), so the velocity vector arrows are getting shorter.
- The particle passes the origin $x = 0$ m at $t \approx -1.5$ s, but it is still moving to the left.
- The position reaches a minimum at $t = -1$ s; the particle is as far left as it is going. The velocity is *instantaneously* $v_x = 0$ m/s as the particle reverses direction.
- The particle moves back to the right between $t = -1$ s and $t = 1$ s ($v_x > 0$).
- The particle turns around again at $t = 1$ s and begins moving back to the left ($v_x < 0$). It keeps speeding up, then disappears off to the left.

A point in the motion where a particle reverses direction is called a **turning point.** It is a point where the velocity is instantaneously zero while the position is a maximum or minimum. This particle has two turning points, at $t = -1$ s and again at $t = +1$ s. We will see many other examples of turning points.

FIGURE 2.13 Motion diagram for Example 2.4.



STOP TO THINK 2.2 Which velocity-versus-time graph goes with the position-versus-time graph on the left?

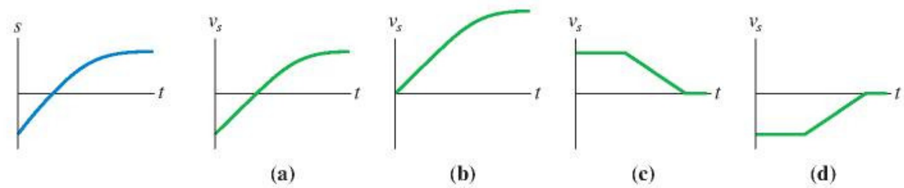
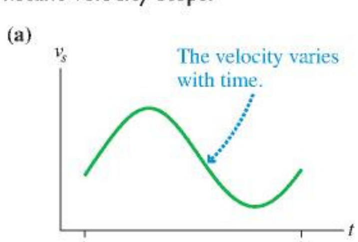
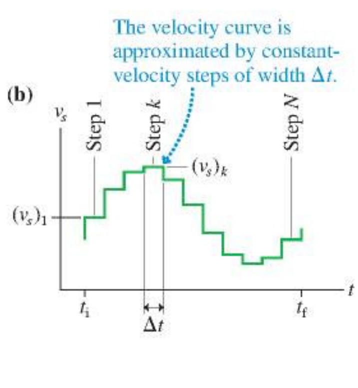


FIGURE 2.14 Approximating a velocity-versus-time graph with a series of constant-velocity steps.





2.3 Finding Position from Velocity

Equation 2.4 allows us to find the instantaneous velocity v_s if we know the position s as a function of time. But what about the reverse problem? Can we use the object's velocity to calculate its position at some future time t? Equation 2.3, $s_f = s_i + v_s \Delta t$, does this for the case of uniform motion with a constant velocity. We need to find a more general expression that is valid when v_s is not constant.

FIGURE 2.14a is a velocity-versus-time graph for an object whose velocity varies with time. Suppose we know the object's position to be s_i at an initial time t_i. Our goal is to find its position s_f at a later time t_f.

Because we know how to handle constant velocities, using Equation 2.3, let's *approximate* the velocity function of Figure 2.14a as a series of constant-velocity steps of width Δt. This is illustrated in **FIGURE 2.14b**. During the first step, from time t_i to time $t_i + \Delta t$, the velocity has the constant value $(v_s)_1$. The velocity during step k has the constant value $(v_s)_k$. Although the approximation shown in the figure is rather rough, with only 11 steps, we can easily imagine that it could be made as accurate as desired by having more and more ever-narrower steps.

The velocity during each step is constant (uniform motion), so we can apply Equation 2.3 to each step. The object's displacement Δs_1 during the first step is simply $\Delta s_1 = (v_s)_1 \Delta t$. The displacement during the second step $\Delta s_2 = (v_s)_2 \Delta t$, and during step k the displacement is $\Delta s_k = (v_s)_k \Delta t$.

The total displacement of the object between t_i and t_f can be approximated as the sum of all the individual displacements during each of the N constant-velocity steps. That is,

$$\Delta s = s_f - s_i \approx \Delta s_1 + \Delta s_2 + \cdots + \Delta s_N = \sum_{k=1}^{N} (v_s)_k \Delta t \qquad (2.9)$$

where Σ (Greek sigma) is the symbol for summation. With a simple rearrangement, the particle's final position is

$$s_f \approx s_i + \sum_{k=1}^{N} (v_s)_k \Delta t \qquad (2.10)$$

Our goal was to use the object's velocity to find its final position s_f. Equation 2.10 nearly reaches that goal, but Equation 2.10 is only approximate because the constant-velocity steps are only an approximation of the true velocity graph. But if we now let $\Delta t \rightarrow 0$, each step's width approaches zero while the total number of steps N approaches infinity. In this limit, the series of steps becomes a perfect replica of the velocity-versus-time graph and Equation 2.10 becomes exact. Thus

$$s_f = s_i + \lim_{\Delta t \to 0} \sum_{k=1}^{N} (v_s)_k \Delta t = s_i + \int_{t_i}^{t_f} v_s \, dt \qquad (2.11)$$

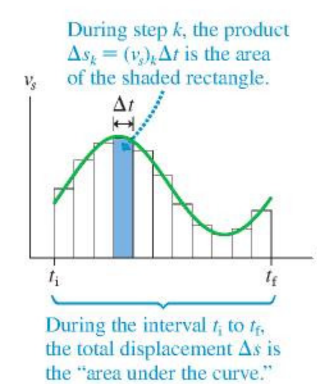
The expression on the right is read, "the integral of $v_s\,dt$ from t_i to t_f." Equation 2.11 is the result that we were seeking. It allows us to predict an object's position s_f at a future time t_f.

We can give Equation 2.11 an important geometric interpretation. **FIGURE 2.15** shows step k in the approximation of the velocity graph as a tall, thin rectangle of height $(v_s)_k$ and width Δt. The product $\Delta s_k = (v_s)_k \Delta t$ is the area (base $\times$ height) of this small rectangle. The sum in Equation 2.11 adds up all of these rectangular areas to give the total area enclosed between the t-axis and the tops of the steps. The limit of this sum as $\Delta t \rightarrow 0$ is the total area enclosed between the t-axis and the velocity curve. This is called the "area under the curve." Thus a graphical interpretation of Equation 2.11 is

$$s_f = s_i + \text{area under the velocity curve } v_s \text{ between } t_i \text{ and } t_f \qquad (2.12)$$

NOTE Wait a minute! The displacement $\Delta s = s_f - s_i$ is a length. How can a length equal an area? Recall earlier, when we found that the velocity is the slope of the position graph, we made a distinction between the *actual* slope and the *physically meaningful* slope? The same distinction applies here. We need to measure the quantities we are using, v_s and Δt, by referring to the scales on the axes. Δt is some number of seconds while v_s is some number of meters per second. When these are multiplied together, the *physically meaningful* area has units of meters.

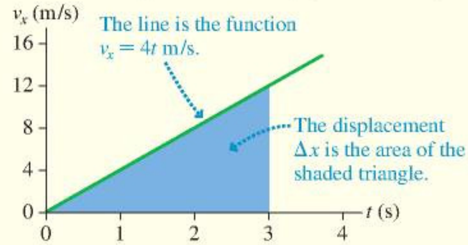
FIGURE 2.15 The total displacement Δs is the "area under the curve."



EXAMPLE 2.5 The displacement during a drag race

FIGURE 2.16 shows the velocity-versus-time graph of a drag racer. How far does the racer move during the first 3.0 s?

FIGURE 2.16 Velocity-versus-time graph for Example 2.5.



MODEL Model the drag racer as a particle with a well-defined position at all times.

VISUALIZE Figure 2.16 is the graphical representation.

SOLVE The question "How far?" indicates that we need to find a displacement Δx rather than a position x. According to Equation 2.12, the car's displacement $\Delta x = x_f - x_i$ between $t = 0$ s and $t = 3$ s is the area under the curve from $t = 0$ s to $t = 3$ s. The curve in this case is an angled line, so the area is that of a triangle:

$$\Delta x = \text{area of triangle between } t = 0 \text{ s and } t = 3 \text{ s}$$
$$= \tfrac{1}{2} \times \text{base} \times \text{height}$$
$$= \tfrac{1}{2} \times 3 \text{ s} \times 12 \text{ m/s} = 18 \text{ m}$$

The drag racer moves 18 m during the first 3 seconds.

ASSESS The "area" is a product of s with m/s, so Δx has the proper units of m.

EXAMPLE 2.6 **Finding the turning point**

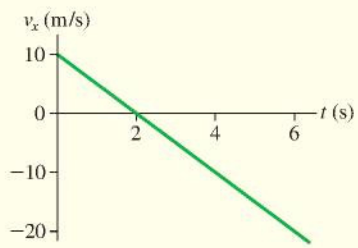
FIGURE 2.17 is the velocity graph for a particle that starts at $x_i = 30$ m at time $t_i = 0$ s.

a. Draw a motion diagram for the particle.

b. Where is the particle's turning point?

c. At what time does the particle reach the origin?

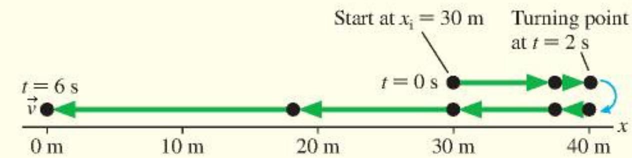
FIGURE 2.17 Velocity-versus-time graph for the particle of Example 2.6.



VISUALIZE The particle is initially 30 m to the right of the origin and moving *to the right* ($v_x > 0$) with a speed of 10 m/s. But v_x is decreasing, so the particle is slowing down. At $t = 2$ s the velocity, just for an instant, is zero before becoming negative. This is the turning point. The velocity is negative for $t > 2$ s, so the particle has reversed direction and moves back toward the origin. At some later time, which we want to find, the particle will pass $x = 0$ m.

SOLVE a. FIGURE 2.18 shows the motion diagram. The distance scale will be established in parts b and c but is shown here for convenience.

FIGURE 2.18 Motion diagram for the particle whose velocity graph was shown in Figure 2.17.



b. The particle reaches the turning point at $t = 2$ s. To learn *where* it is at that time we need to find the displacement during the first two seconds. We can do this by finding the area under the curve between $t = 0$ s and $t = 2$ s:

$$\begin{aligned} x(\text{at } t = 2\text{ s}) &= x_i + \text{area under the curve between 0 s and 2 s} \\ &= 30\text{ m} + \tfrac{1}{2}(2\text{ s} - 0\text{ s})(10\text{ m/s} - 0\text{ m/s}) \\ &= 40\text{ m} \end{aligned}$$

The turning point is at $x = 40$ m.

c. The particle needs to move $\Delta x = -40$ m to get from the turning point to the origin. That is, the area under the curve from $t = 2$ s to the desired time t needs to be -40 m. Because the curve is below the axis, with negative values of v_x, the area to the right of $t = 2$ s is a *negative* area. With a bit of geometry, you will find that the triangle with a base extending from $t = 2$ s to $t = 6$ s has an area of -40 m. Thus the particle reaches the origin at $t = 6$ s.

A Little More Calculus: Integrals

Taking the derivative of a function is equivalent to finding the slope of a graph of the function. Similarly, evaluating an integral is equivalent to finding the area under a graph of the function. The graphical method is very important for building intuition about motion but is limited in its practical application. Just as derivatives of standard functions can be evaluated and tabulated, so can integrals.

The integral in Equation 2.11 is called a *definite integral* because there are two definite boundaries to the area we want to find. These boundaries are called the lower (t_i) and upper (t_f) *limits of integration*. For the important function $u(t) = ct^n$, the essential result from calculus is that

$$\int_{t_i}^{t_f} u\,dt = \int_{t_i}^{t_f} ct^n\,dt = \frac{ct^{n+1}}{n+1}\bigg|_{t_i}^{t_f} = \frac{ct_f^{n+1}}{n+1} - \frac{ct_i^{n+1}}{n+1} \qquad (n \neq -1) \tag{2.13}$$

The vertical bar in the third step with subscript t_i and superscript t_f is a shorthand notation from calculus that means—as seen in the last step—the integral evaluated at the upper limit t_f *minus* the integral evaluated at the lower limit t_i. You also need to know that for two functions u and w,

$$\int_{t_i}^{t_f} (u + w)\,dt = \int_{t_i}^{t_f} u\,dt + \int_{t_i}^{t_f} w\,dt \tag{2.14}$$

That is, the integral of a sum is equal to the sum of the integrals.

EXAMPLE 2.7 **Using calculus to find the position**

Use calculus to solve Example 2.6.

SOLVE Figure 2.17 is a linear graph. Its "y-intercept" is seen to be 10 m/s and its slope is -5 (m/s)/s. Thus the velocity can be described by the equation

$$v_x = (10 - 5t) \text{ m/s}$$

where t is in s. We can find the position x at time t by using Equation 2.11:

$$x = x_i + \int_0^t v_x\,dt = 30 \text{ m} + \int_0^t (10 - 5t)\,dt$$
$$= 30 \text{ m} + \int_0^t 10\,dt - \int_0^t 5t\,dt$$

We used Equation 2.14 for the integral of a sum to get the final expression. The first integral is a function of the form $u = ct^n$ with $c = 10$ and $n = 0$; the second is of the form $u = ct^n$ with $c = 5$ and $n = 1$. Using Equation 2.13, we have

$$\int_0^t 10\,dt = 10t\Big|_0^t = 10 \cdot t - 10 \cdot 0 = 10t \text{ m}$$

and

$$\int_0^t 5t\,dt = \tfrac{5}{2}t^2\Big|_0^t = \tfrac{5}{2} \cdot t^2 - \tfrac{5}{2} \cdot 0^2 = \tfrac{5}{2}t^2 \text{ m}$$

Combining the pieces gives

$$x = \left(30 + 10t - \tfrac{5}{2}t^2\right) \text{ m}$$

This is a general result for the position at *any* time t.

The particle's turning point occurs at $t = 2$ s, and its position at that time is

$$x(\text{at } t = 2 \text{ s}) = 30 + (10)(2) - \tfrac{5}{2}(2)^2 = 40 \text{ m}$$

The time at which the particle reaches the origin is found by setting $x = 0$ m:

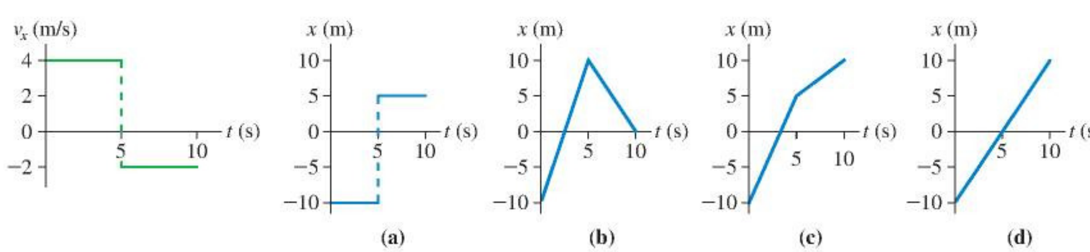
$$30 + 10t - \tfrac{5}{2}t^2 = 0$$

This quadratic equation has two solutions: $t = -2$ s or $t = 6$ s.

When we solve a quadratic equation, we cannot just arbitrarily select the root we want. Instead, we must decide which is the *meaningful* root. Here the negative root refers to a time before the problem began, so the meaningful one is the positive root, $t = 6$ s.

ASSESS The results agree with the answers we found previously from a graphical solution.

STOP TO THINK 2.3 Which position-versus-time graph goes with the velocity-versus-time graph on the left? The particle's position at $t_i = 0$ s is $x_i = -10$ m.



2.4 Motion with Constant Acceleration

We need one more major concept to describe one-dimensional motion: acceleration. Acceleration, as we noted in Chapter 1, is a rather abstract concept. Nonetheless, acceleration is the linchpin of mechanics. We will see very shortly that Newton's laws relate the acceleration of an object to the forces that are exerted on it.

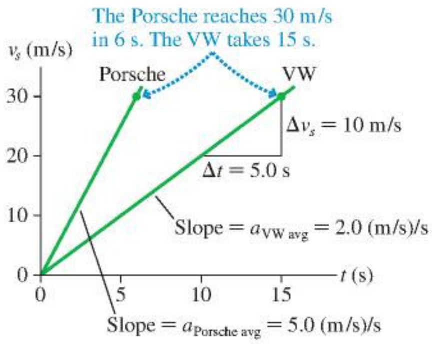
Let's conduct a race between a Volkswagen Beetle and a Porsche to see which can achieve a velocity of 30 m/s ($\approx$60 mph) in the shortest time. Both cars are equipped with computers that will record the speedometer reading 10 times each second. This gives a nearly continuous record of the *instantaneous* velocity of each car. **TABLE 2.1** shows some of the data. The velocity-versus-time graphs, based on these data, are shown in **FIGURE 2.19** on the next page.

How can we describe the difference in performance of the two cars? It is not that one has a different velocity from the other; both achieve every velocity between 0 and 30 m/s. The distinction is how long it took each to *change* its velocity from 0 to 30 m/s. The Porsche changed velocity quickly, in 6.0 s, while the VW needed 15 s to make

TABLE 2.1 Velocities of a Porsche and a Volkswagen Beetle

t(s)	v_{Porsche} (m/s)	v_{VW} (m/s)
0.0	0.0	0.0
0.1	0.5	0.2
0.2	1.0	0.4
0.3	1.5	0.6
⋮	⋮	⋮

FIGURE 2.19 Velocity-versus-time graphs for the Porsche and the VW Beetle.



the same velocity change. Because the Porsche had a velocity change $\Delta v_s = 30$ m/s during a time interval $\Delta t = 6.0$ s, the *rate* at which its velocity changed was

$$\text{rate of velocity change} = \frac{\Delta v_s}{\Delta t} = \frac{30\text{ m/s}}{6.0\text{ s}} = 5.0\text{ (m/s)/s} \tag{2.15}$$

Notice the units. They are units of "velocity per second." A rate of velocity change of 5.0 "meters per second per second" means that the velocity increases by 5.0 m/s during the first second, by another 5.0 m/s during the next second, and so on. In fact, the velocity will increase by 5.0 m/s during any second in which it is changing at the rate of 5.0 (m/s)/s.

Chapter 1 introduced *acceleration* as "the rate of change of velocity." That is, acceleration measures how quickly or slowly an object's velocity changes. In parallel with our treatment of velocity, let's define the **average acceleration** a_{avg} during the time interval Δt to be

$$a_{\text{avg}} \equiv \frac{\Delta v_s}{\Delta t} \quad \text{(average acceleration)} \tag{2.16}$$

Equations 2.15 and 2.16 show that the Porsche had the rather large acceleration of 5.0 (m/s)/s.

Because Δv_s and Δt are the "rise" and "run" of a velocity-versus-time graph, we see that a_{avg} can be interpreted graphically as the *slope* of a straight-line velocity-versus-time graph. In other words,

$$a_{\text{avg}} = \text{slope of the velocity-versus-time graph} \tag{2.17}$$

Figure 2.19 uses this idea to show that the VW's average acceleration is

$$a_{\text{VW avg}} = \frac{\Delta v_s}{\Delta t} = \frac{10\text{ m/s}}{5.0\text{ s}} = 2.0\text{ (m/s)/s}$$

This is less than the acceleration of the Porsche, as expected.

An object whose velocity-versus-time graph is a straight-line graph has a steady and unchanging acceleration. There's no need to specify "average" if the acceleration is constant, so we'll use the symbol a_s as we discuss motion along the s-axis with constant acceleration.

Signs and Units

An important aspect of acceleration is its *sign*. Acceleration $\vec{a}$, like position $\vec{r}$ and velocity $\vec{v}$, is a vector. For motion in one dimension, the sign of a_x (or a_y) is positive if the vector $\vec{a}$ points to the right (or up), negative if it points to the left (or down). This was illustrated in « Figure 1.18 and the very important « Tactics Box 1.4, which you may wish to review. It's particularly important to emphasize that positive and negative values of a_s do *not* correspond to "speeding up" and "slowing down."

EXAMPLE 2.8 **Relating acceleration to velocity**

a. A bicyclist has a velocity of 6 m/s and a constant acceleration of 2 (m/s)/s. What is her velocity 1 s later? 2 s later?

b. A bicyclist has a velocity of −6 m/s and a constant acceleration of 2 (m/s)/s. What is his velocity 1 s later? 2 s later?

SOLVE

a. An acceleration of 2 (m/s)/s *means* that the velocity increases by 2 m/s every 1 s. If the bicyclist's initial velocity is 6 m/s, then 1 s later her velocity will be 8 m/s. After 2 s, which is 1 additional second later, it will increase by another 2 m/s to 10 m/s. After 3 s it will be 12 m/s. Here a positive a_x is causing the bicyclist to speed up.

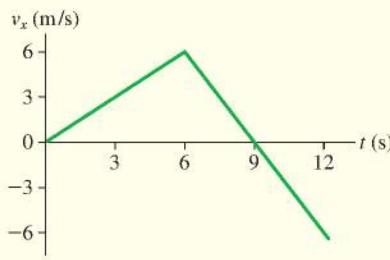
b. If the bicyclist's initial velocity is a *negative* −6 m/s but the acceleration is a positive +2 (m/s)/s, then 1 s later his velocity will be −4 m/s. After 2 s it will be −2 m/s, and so on. In this case, a positive a_x is causing the object to *slow down* (decreasing speed v). This agrees with the rule from Tactics Box 1.4: An object is slowing down if and only if v_x and a_x have opposite signs.

NOTE It is customary to abbreviate the acceleration units (m/s)/s as m/s². For example, the bicyclists in Example 2.8 had an acceleration of 2 m/s². We will use this notation, but keep in mind the *meaning* of the notation as "(meters per second) per second."

EXAMPLE 2.9 Running the court

A basketball player starts at the left end of the court and moves with the velocity shown in FIGURE 2.20. Draw a motion diagram and an acceleration-versus-time graph for the basketball player.

FIGURE 2.20 Velocity-versus-time graph for the basketball player of Example 2.9.



VISUALIZE The velocity is positive (motion to the right) and increasing for the first 6 s, so the velocity arrows in the motion diagram are to the right and getting longer. From $t = 6$ s to 9 s the motion is still to the right (v_x is still positive), but the arrows are getting shorter because v_x is decreasing. There's a turning point at $t = 9$ s, when $v_x = 0$, and after that the motion is to the left (v_x is negative) and getting faster. The motion diagram of FIGURE 2.21a shows the velocity and the acceleration vectors.

SOLVE Acceleration is the slope of the velocity graph. For the first 6 s, the slope has the constant value

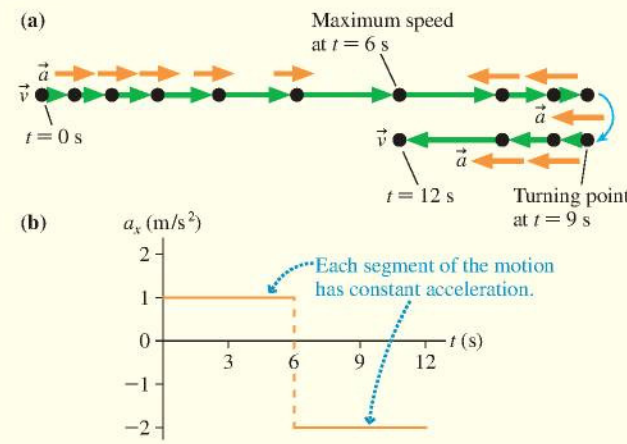
$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{6.0 \text{ m/s}}{6.0 \text{ s}} = 1.0 \text{ m/s}^2$$

The velocity then decreases by 12 m/s during the 6 s interval from $t = 6$ s to $t = 12$ s, so

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{-12 \text{ m/s}}{6.0 \text{ s}} = -2.0 \text{ m/s}^2$$

The acceleration graph for these 12 s is shown in FIGURE 2.21b. Notice that there is no change in the acceleration at $t = 9$ s, the turning point.

FIGURE 2.21 Motion diagram and acceleration graph for Example 2.9.



ASSESS The *sign* of a_x does *not* tell us whether the object is speeding up or slowing down. The basketball player is slowing down from $t = 6$ s to $t = 9$ s, then speeding up from $t = 9$ s to $t = 12$ s. Nonetheless, his acceleration is negative during this entire interval because his acceleration vector, as seen in the motion diagram, always points to the left.

The Kinematic Equations of Constant Acceleration

Consider an object whose acceleration a_s remains constant during the time interval $\Delta t = t_f - t_i$. At the beginning of this interval, at time t_i, the object has initial velocity v_{is} and initial position s_i. Note that t_i is often zero, but it does not have to be. We would like to predict the object's final position s_f and final velocity v_{fs} at time t_f.

The object's velocity is changing because the object is accelerating. FIGURE 2.22a shows the acceleration-versus-time graph, a horizontal line between t_i and t_f. It is not hard to find the object's velocity v_{fs} at a later time t_f. By definition,

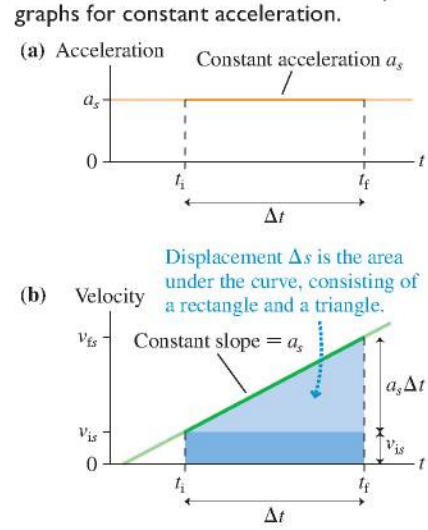
$$a_s = \frac{\Delta v_s}{\Delta t} = \frac{v_{fs} - v_{is}}{\Delta t} \tag{2.18}$$

which is easily rearranged to give

$$v_{fs} = v_{is} + a_s \Delta t \tag{2.19}$$

The velocity-versus-time graph, shown in FIGURE 2.22b, is a straight line that starts at v_{is} and has slope a_s.

FIGURE 2.22 Acceleration and velocity graphs for constant acceleration.



As you learned in the last section, the object's final position is

$$s_f = s_i + \text{area under the velocity curve } v_s \text{ between } t_i \text{ and } t_f \tag{2.20}$$

The shaded area in Figure 2.22b can be subdivided into a rectangle of area $v_{is}\,\Delta t$ and a triangle of area $\frac{1}{2}(a_s\,\Delta t)(\Delta t) = \frac{1}{2}a_s(\Delta t)^2$. Adding these gives

$$s_f = s_i + v_{is}\,\Delta t + \tfrac{1}{2}a_s(\Delta t)^2 \tag{2.21}$$

where $\Delta t = t_f - t_i$ is the elapsed time. The quadratic dependence on Δt causes the position-versus-time graph for constant-acceleration motion to have a parabolic shape, as shown in Model 2.2.

Equations 2.19 and 2.21 are two of the basic kinematic equations for motion with *constant* acceleration. They allow us to predict an object's position and velocity at a future instant of time. We need one more equation to complete our set, a direct relation between position and velocity. First use Equation 2.19 to write $\Delta t = (v_{fs} - v_{is})/a_s$. Substitute this into Equation 2.21, giving

$$s_f = s_i + v_{is}\left(\frac{v_{fs} - v_{is}}{a_s}\right) + \tfrac{1}{2}a_s\left(\frac{v_{fs} - v_{is}}{a_s}\right)^2 \tag{2.22}$$

With a bit of algebra, this is rearranged to read

$$v_{fs}^2 = v_{is}^2 + 2a_s\,\Delta s \tag{2.23}$$

where $\Delta s = s_f - s_i$ is the *displacement* (not the distance!). Equation 2.23 is the last of the three kinematic equations for motion with constant acceleration.

The Constant-Acceleration Model

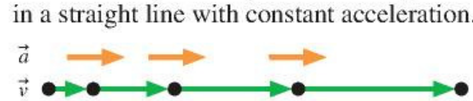
Few objects with changing velocity have a perfectly constant acceleration, but it is often reasonable to model their acceleration as being constant. We do so by utilizing the **constant-acceleration model.** Once again, a model is a set of words, pictures, graphs, and equations that allows us to explain and predict an object's motion.

MODEL 2.2 MP

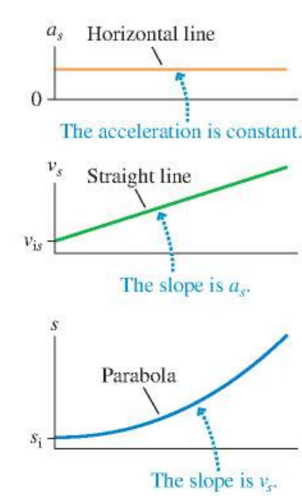
Constant acceleration

For motion with constant acceleration.

- Model the object as a particle moving in a straight line with constant acceleration.



- Mathematically:
  - $v_{fs} = v_{is} + a_s\,\Delta t$
  - $s_f = s_i + v_{is}\,\Delta t + \frac{1}{2}a_s(\Delta t)^2$
  - $v_{fs}^2 = v_{is}^2 + 2a_s\,\Delta s$
- Limitations: Model fails if the particle's acceleration changes.



Exercise 16

In this text, we'll usually model runners, cars, planes, and rockets as having constant acceleration. Their actual acceleration is often more complicated (for example, a car's acceleration gradually decreases rather than remaining constant until full speed is reached), but the mathematical complexity of dealing with realistic accelerations would detract from the physics we're trying to learn.

The constant-acceleration model is the basis for a problem-solving strategy.

PROBLEM-SOLVING STRATEGY 2.1

Kinematics with constant acceleration

MODEL Model the object as having constant acceleration.

VISUALIZE Use different representations of the information in the problem.

- Draw a *pictorial representation.* This helps you assess the information you are given and starts the process of translating the problem into symbols.
- Use a *graphical representation* if it is appropriate for the problem.
- Go back and forth between these two representations as needed.

SOLVE The mathematical representation is based on the three kinematic equations:

$$v_{fs} = v_{is} + a_s \Delta t$$
$$s_f = s_i + v_{is}\,\Delta t + \tfrac{1}{2}a_s(\Delta t)^2$$
$$v_{fs}^{\,2} = v_{is}^{\,2} + 2a_s\,\Delta s$$

- Use x or y, as appropriate to the problem, rather than the generic s.
- Replace i and f with numerical subscripts defined in the pictorial representation.

ASSESS Check that your result has the correct units and significant figures, is reasonable, and answers the question.

NOTE You are strongly encouraged to solve problems on the Dynamics Worksheets found at the back of the Student Workbook. These worksheets will help you use the Problem-Solving Strategy and develop good problem-solving skills.

EXAMPLE 2.10 The motion of a rocket sled

A rocket sled's engines fire for 5.0 s, boosting the sled to a speed of 250 m/s. The sled then deploys a braking parachute, slowing by 3.0 m/s per second until it stops. What is the total distance traveled?

MODEL We're not given the sled's initial acceleration, while the rockets are firing, but rocket sleds are aerodynamically shaped to minimize air resistance and so it seems reasonable to model the sled as a particle undergoing constant acceleration.

VISUALIZE FIGURE 2.23 shows the pictorial representation. We've made the reasonable assumptions that the sled starts from rest and that the braking parachute is deployed just as the rocket burn ends. There are three points of interest in this problem: the start, the change from propulsion to braking, and the stop. Each of these points has been assigned a position, velocity, and time. Notice that we've replaced the generic subscripts i and f of the kinematic equations with the numerical subscripts 0, 1, and 2. Accelerations are associated not with specific points in the motion but with the intervals between the points, so acceleration a_{0x} is the acceleration between points 0 and 1 while acceleration a_{1x} is the acceleration between points 1 and 2. The acceleration vector $\vec{a}_1$ points to the left, so a_{1x} is negative. The sled stops at the end point, so $v_{2x} = 0$ m/s.

SOLVE We know how long the rocket burn lasts and the velocity at the end of the burn. Because we're modeling the sled as having uniform acceleration, we can use the first kinematic equation of Problem-Solving Strategy 2.1 to write

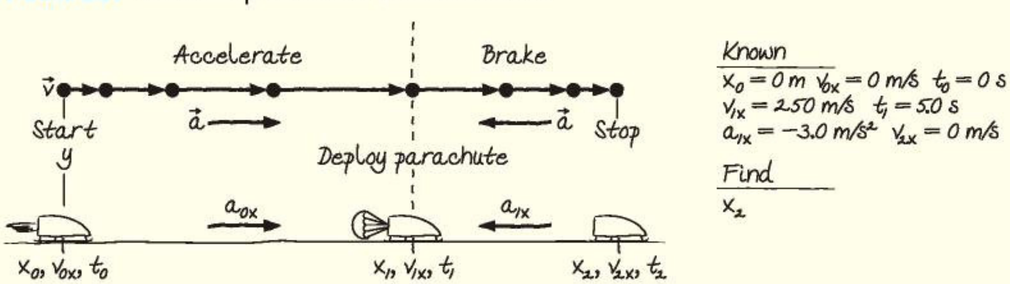
$$v_{1x} = v_{0x} + a_{0x}(t_1 - t_0) = a_{0x}t_1$$

We started with the complete equation, then simplified by noting which terms were zero. Solving for the boost-phase acceleration, we have

$$a_{0x} = \frac{v_{1x}}{t_1} = \frac{250 \text{ m/s}}{5.0 \text{ s}} = 50 \text{ m/s}^2$$

Notice that we worked algebraically until the last step—a hallmark of good problem-solving technique that minimizes the chances of

FIGURE 2.23 Pictorial representation of the rocket sled.



Continued

calculation errors. Also, in accord with the significant figure rules of Chapter 1, 50 m/s^2 is considered to have two significant figures.

Now we have enough information to find out how far the sled travels while the rockets are firing. The second kinematic equation of Problem-Solving Strategy 2.1 is

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_{0x}(t_1 - t_0)^2 = \tfrac{1}{2}a_{0x}t_1^2$$
$$= \tfrac{1}{2}(50\text{ m/s}^2)(5.0\text{ s})^2 = 625\text{ m}$$

The braking phase is a little different because we don't know how long it lasts. But we do know both the initial and final velocities, so we can use the third kinematic equation of Problem-Solving Strategy 2.1:

$$v_{2x}^2 = v_{1x}^2 + 2a_{1x}\Delta x = v_{1x}^2 + 2a_{1x}(x_2 - x_1)$$

Notice that Δx is *not* x_2; it's the displacement $(x_2 - x_1)$ during the braking phase. We can now solve for x_2:

$$x_2 = x_1 + \frac{v_{2x}^2 - v_{1x}^2}{2a_{1x}}$$
$$= 625\text{ m} + \frac{0 - (250\text{ m/s})^2}{2(-3.0\text{ m/s}^2)} = 11{,}000\text{ m}$$

We kept three significant figures for x_1 at an intermediate stage of the calculation but rounded to two significant figures at the end.

ASSESS The total distance is 11 km ≈ 7 mi. That's large but believable. Using the approximate conversion factor 1 m/s ≈ 2 mph from Table 1.5, we see that the top speed is ≈ 500 mph. It will take a long distance for the sled to gradually stop from such a high speed.

EXAMPLE 2.11 A two-car race

Fred is driving his Volkswagen Beetle at a steady 20 m/s when he passes Betty sitting at rest in her Porsche. Betty instantly begins accelerating at 5.0 m/s^2. How far does Betty have to drive to overtake Fred?

MODEL Model the VW as a particle in uniform motion and the Porsche as a particle with constant acceleration.

VISUALIZE FIGURE 2.24 is the pictorial representation. Fred's motion diagram is one of uniform motion, while Betty's shows uniform acceleration. Fred is ahead in frames 1, 2, and 3, but Betty catches up with him in frame 4. The coordinate system shows the cars with the same position at the start and at the end—but with the important difference that Betty's Porsche has an acceleration while Fred's VW does not.

SOLVE This problem is similar to Example 2.2, in which Bob and Susan met for lunch. As we did there, we want to find Betty's position $(x_1)_B$ at the instant t_1 when $(x_1)_B = (x_1)_F$. We know, from the models of uniform motion and uniform acceleration, that Fred's position graph is a straight line but Betty's is a parabola. The position graphs in Figure 2.24 show that we're solving for the intersection point of the line and the parabola.

Fred's and Betty's positions at t_1 are

$$(x_1)_F = (x_0)_F + (v_{0x})_F(t_1 - t_0) = (v_{0x})_F t_1$$
$$(x_1)_B = (x_0)_B + (v_{0x})_B(t_1 - t_0) + \tfrac{1}{2}(a_{0x})_B(t_1 - t_0)^2 = \tfrac{1}{2}(a_{0x})_B t_1^2$$

By equating these,

$$(v_{0x})_F t_1 = \tfrac{1}{2}(a_{0x})_B t_1^2$$

we can solve for the time when Betty passes Fred:

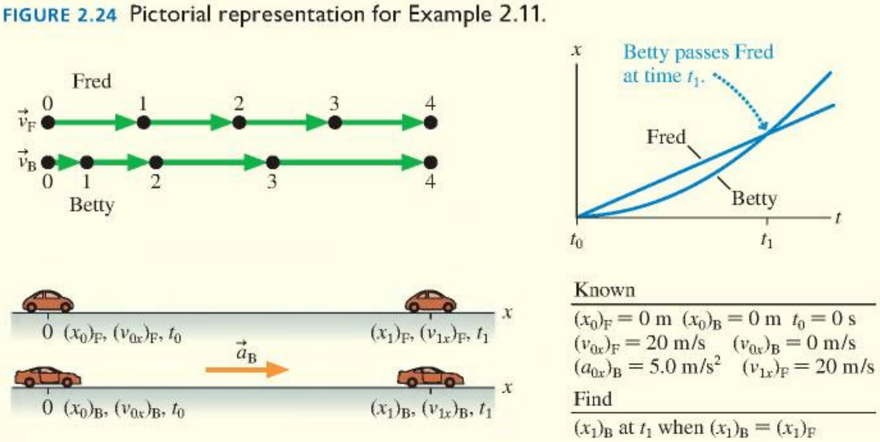
$$t_1\left[\tfrac{1}{2}(a_{0x})_B t_1 - (v_{0x})_F\right] = 0$$
$$t_1 = \begin{cases} 0\text{ s} \\ 2(v_{0x})_F/(a_{0x})_B = 8.0\text{ s} \end{cases}$$

Interestingly, there are two solutions. That's not surprising, when you think about it, because the line and the parabola of the position graphs have *two* intersection points: when Fred first passes Betty, and 8.0 s later when Betty passes Fred. We're interested in only the second of these points. We can now use either of the distance equations to find $(x_1)_B = (x_1)_F = 160$ m. Betty has to drive 160 m to overtake Fred.

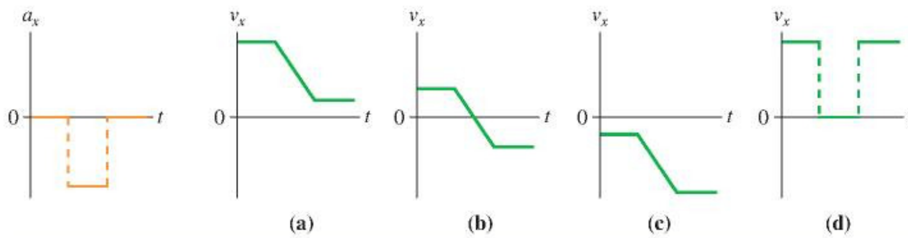
ASSESS 160 m ≈ 160 yards. Because Betty starts from rest while Fred is moving at 20 m/s ≈ 40 mph, needing 160 yards to catch him seems reasonable.

NOTE The purpose of the Assess step is not to prove that an answer must be right but to rule out answers that, with a little thought, are clearly wrong.

FIGURE 2.24 Pictorial representation for Example 2.11.



STOP TO THINK 2.4 Which velocity-versus-time graph or graphs go with the acceleration-versus-time graph on the left? The particle is initially moving to the right.



2.5 Free Fall

The motion of an object moving under the influence of gravity only, and no other forces, is called **free fall.** Strictly speaking, free fall occurs only in a vacuum, where there is no air resistance. Fortunately, the effect of air resistance is small for "heavy objects," so we'll make only a very slight error in treating these objects *as if* they were in free fall. For very light objects, such as a feather, or for objects that fall through very large distances and gain very high speeds, the effect of air resistance is *not* negligible. Motion with air resistance is a problem we will study in Chapter 6. Until then, we will restrict our attention to "heavy objects" and will make the reasonable assumption that falling objects are in free fall.

Galileo, in the 17th century, was the first to make detailed measurements of falling objects. The story of Galileo dropping different weights from the leaning bell tower at the cathedral in Pisa is well known, although historians cannot confirm its truth. Based on his measurements, wherever they took place, Galileo developed a *model* for motion in the absence of air resistance:

- Two objects dropped from the same height will, if air resistance can be neglected, hit the ground at the same time and with the same speed.
- Consequently, **any two objects in free fall, regardless of their mass, have the same acceleration $\vec{a}_{\text{free fall}}$.**

In a vacuum, the apple and feather fall at the same rate and hit the ground at the same time.

FIGURE 2.25a shows the motion diagram of an object that was released from rest and falls freely. FIGURE 2.25b shows the object's velocity graph. The motion diagram and graph are identical for a falling pebble and a falling boulder. The fact that the velocity graph is a straight line tells us the motion is one of constant acceleration, and $a_{\text{free fall}}$ is found from the slope of the graph. Careful measurements show that the value of $\vec{a}_{\text{free fall}}$ varies ever so slightly at different places on the earth, due to the slightly nonspherical shape of the earth and to the fact that the earth is rotating. A global average, at sea level, is

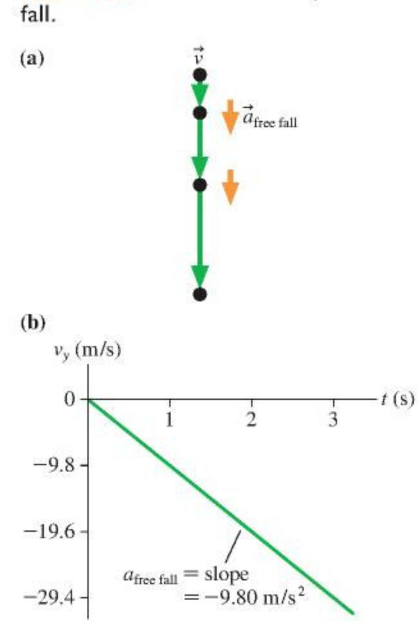
$$\vec{a}_{\text{free fall}} = (9.80 \text{ m/s}^2, \text{ vertically downward}) \tag{2.24}$$

Vertically downward means along a line toward the center of the earth.

The length, or magnitude, of $\vec{a}_{\text{free fall}}$ is known as the **free-fall acceleration,** and it has the special symbol g:

$$g = 9.80 \text{ m/s}^2 \text{ (free-fall acceleration)}$$

FIGURE 2.25 Motion of an object in free fall.



Several points about free fall are worthy of note:

- g, by definition, is *always* positive. **There will never be a problem that will use a negative value for g.** But, you say, objects fall when you release them rather than rise, so how can g be positive?
- g is *not* the acceleration $a_{\text{free fall}}$, but simply its magnitude. Because we've chosen the y-axis to point vertically upward, the downward acceleration vector $\vec{a}_{\text{free fall}}$ has the one-dimensional acceleration

$$a_y = a_{\text{free fall}} = -g \tag{2.25}$$

It is a_y that is negative, not g.

- We can model free fall as motion with constant acceleration, with $a_y = -g$.
- g is not called "gravity." Gravity is a force, not an acceleration. The symbol g recognizes the influence of gravity, but g is *the free-fall acceleration.*
- $g = 9.80\ \text{m/s}^2$ only on earth. Other planets have different values of g. You will learn in Chapter 13 how to determine g for other planets.

NOTE Despite the name, free fall is not restricted to objects that are literally falling. Any object moving under the influence of gravity only, and no other forces, is in free fall. This includes objects falling straight down, objects that have been tossed or shot straight up, and projectile motion.

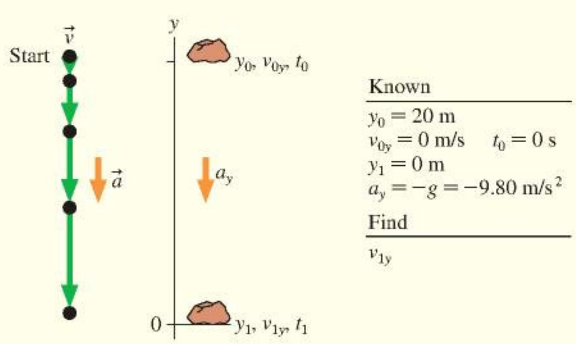
EXAMPLE 2.12 A falling rock

A rock is dropped from the top of a 20-m-tall building. What is its impact velocity?

MODEL A rock is fairly heavy, and air resistance is probably not a serious concern in a fall of only 20 m. It seems reasonable to model the rock's motion as free fall: constant acceleration with $a_y = a_{\text{free fall}} = -g$.

VISUALIZE FIGURE 2.26 shows the pictorial representation. We have placed the origin at the ground, which makes $y_0 = 20\ \text{m}$. Although the rock falls 20 m, it is important to notice that the *displacement* is $\Delta y = y_1 - y_0 = -20\ \text{m}$.

FIGURE 2.26 Pictorial representation of a falling rock.



SOLVE In this problem we know the displacement but not the time, which suggests that we use the third kinematic equation from Problem-Solving Strategy 2.1:

$$v_{1y}{}^2 = v_{0y}{}^2 + 2a_y\,\Delta y = -2g\,\Delta y$$

We started by writing the general equation, then noted that $v_{0y} = 0$ m/s and substituted $a_y = -g$. Solving for v_{1y}:

$$v_{1y} = \sqrt{-2g\Delta y} = \sqrt{-2(9.8\ \text{m/s}^2)(-20\ \text{m})} = \pm 20\ \text{m/s}$$

A common error would be to say, "The rock fell 20 m, so $\Delta y = 20$ m." That would have you trying to take the square root of a negative number. As noted above, Δy is a *displacement,* not a distance, and in this case $\Delta y = -20$ m.

The $\pm$ sign indicates that there are two mathematical solutions; therefore, we have to use physical reasoning to choose between them. The rock does hit with a *speed* of 20 m/s, but the question asks for the impact *velocity*. The velocity vector points down, so the sign of v_{1y} is negative. Thus the impact velocity is -20 m/s.

ASSESS Is the answer reasonable? Well, 20 m is about 60 feet, or about the height of a five- or six-story building. Using 1 m/s $\approx$ 2 mph, we see that 20 m/s $\approx$ 40 mph. That seems quite reasonable for the speed of an object after falling five or six stories. If we had misplaced a decimal point, though, and found 2.0 m/s, we would be suspicious that this was much too small after converting it to $\approx$ 4 mph.

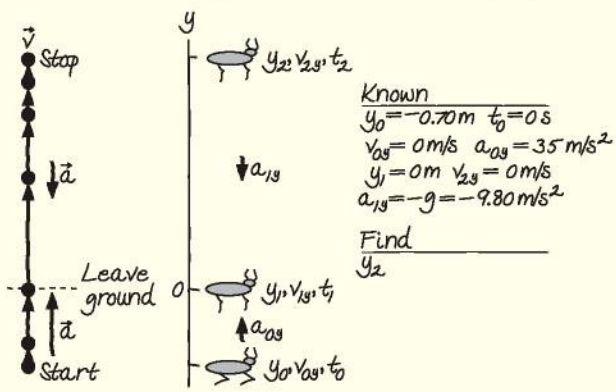
EXAMPLE 2.13 Finding the height of a leap

The springbok, an antelope found in Africa, gets its name from its remarkable jumping ability. When startled, a springbok will leap straight up into the air—a maneuver called a "pronk." A springbok goes into a crouch to perform a pronk. It then extends its legs forcefully, accelerating at 35 m/s^2 for 0.70 m as its legs straighten. Legs fully extended, it leaves the ground and rises into the air. How high does it go?

MODEL The springbok is changing shape as it leaps, so can we reasonably model it as a particle? We can if we focus on the *body* of the springbok, treating the expanding legs like external springs. Initially, the body of the springbok is driven upward by its legs. We'll model this as a particle—the body—undergoing constant acceleration. Once the springbok's feet leave the ground, we'll model the motion of the springbok's body as a particle in free fall.

VISUALIZE FIGURE 2.27 shows the pictorial representation. This is a problem with a beginning point, an end point, and a point in between where the nature of the motion changes. We've identified these points with subscripts 0, 1, and 2. The motion from 0 to 1 is a rapid upward acceleration until the springbok's feet leave the ground at 1. Even though the springbok is moving upward from 1 to 2, this is free-fall motion because the springbok is now moving under the influence of gravity only.

FIGURE 2.27 Pictorial representation of a startled springbok.



How do we put "How high?" into symbols? The clue is that the very top point of the trajectory is a *turning point*, and we've seen that the instantaneous velocity at a turning point is $v_{2y} = 0$. This was not explicitly stated but is part of our interpretation of the problem.

SOLVE For the first part of the motion, pushing off, we know a displacement but not a time interval. We can use

$$v_{1y}{}^2 = v_{0y}{}^2 + 2a_{0y}\,\Delta y = 2(35 \text{ m/s}^2)(0.70 \text{ m}) = 49 \text{ m}^2/\text{s}^2$$

$$v_{1y} = \sqrt{49 \text{ m}^2/\text{s}^2} = 7.0 \text{ m/s}$$

The springbok leaves the ground with a velocity of 7.0 m/s. This is the starting point for the problem of a projectile launched straight up from the ground. One possible solution is to use the velocity equation to find how long it takes to reach maximum height, then the position equation to calculate the maximum height. But that takes two separate calculations. It is easier to make another use of the velocity-displacement equation:

$$v_{2y}{}^2 = 0 = v_{1y}{}^2 + 2a_{1y}\,\Delta y = v_{1y}{}^2 - 2g(y_2 - y_1)$$

where now the acceleration is $a_{1y} = -g$. Using $y_1 = 0$, we can solve for y_2, the height of the leap:

$$y_2 = \frac{v_{1y}{}^2}{2g} = \frac{(7.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 2.5 \text{ m}$$

ASSESS 2.5 m is a bit over 8 feet, a remarkable vertical jump. But these animals are known for their jumping ability, so the answer seems reasonable. Note that it is especially important in a multipart problem like this to use numerical subscripts to distinguish different points in the motion.

2.6 Motion on an Inclined Plane

FIGURE 2.28a shows a problem closely related to free fall: that of motion down a straight, but frictionless, inclined plane, such as a skier going down a slope on frictionless snow. What is the object's acceleration? Although we're not yet prepared to give a rigorous derivation, we can deduce the acceleration with a plausibility argument.

FIGURE 2.28 Acceleration on an inclined plane.

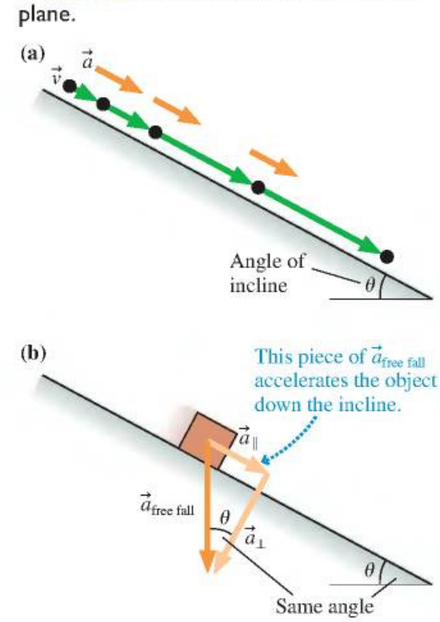


FIGURE 2.28b shows the free-fall acceleration $\vec{a}_{\text{free fall}}$ the object would have if the incline suddenly vanished. The free-fall acceleration points straight down. This vector can be broken into two pieces: a vector $\vec{a}_\parallel$ that is parallel to the incline and a vector $\vec{a}_\perp$ that is perpendicular to the incline. The surface of the incline somehow "blocks" $\vec{a}_\perp$, through a process we will examine in Chapter 6, but $\vec{a}_\parallel$ is unhindered. It is this piece of $\vec{a}_{\text{free fall}}$, parallel to the incline, that accelerates the object.

By definition, the length, or magnitude, of $\vec{a}_{\text{free fall}}$ is g. Vector $\vec{a}_\parallel$ is opposite angle θ (Greek *theta*), so the length, or magnitude, of $\vec{a}_\parallel$ must be $g \sin\theta$. Consequently, the one-dimensional acceleration along the incline is

$$a_s = \pm g \sin\theta \qquad (2.26)$$

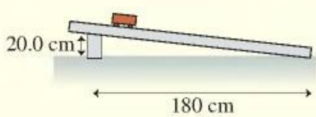
The correct sign depends on the direction in which the ramp is tilted. Examples will illustrate.

Equation 2.26 makes sense. Suppose the plane is perfectly horizontal. If you place an object on a horizontal surface, you expect it to stay at rest with no acceleration. Equation 2.26 gives $a_s = 0$ when $\theta = 0°$, in agreement with our expectations. Now suppose you tilt the plane until it becomes vertical, at $\theta = 90°$. Without friction, an object would simply fall, in free fall, parallel to the vertical surface. Equation 2.26 gives $a_s = -g = a_{\text{free fall}}$ when $\theta = 90°$, again in agreement with our expectations. Equation 2.26 gives the correct result in these *limiting cases*.

EXAMPLE 2.14 Measuring acceleration

In the laboratory, a 2.00-m-long track has been inclined as shown in FIGURE 2.29. Your task is to measure the acceleration of a cart on the ramp and to compare your result with what you might have expected. You have available five "photogates" that measure the cart's speed as it passes through. You place a gate every 30 cm from a line you mark near the top of the track as the starting line. One run generates the data shown in the table. The first entry isn't a photogate, but it is a valid data point because you know the cart's speed is zero at the point where you release it.

FIGURE 2.29 The experimental setup.



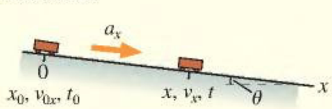
Distance (cm)	Speed (m/s)
0	0.00
30	0.75
60	1.15
90	1.38
120	1.56
150	1.76

NOTE Physics is an experimental science. Our knowledge of the universe is grounded in observations and measurements. Consequently, some examples and homework problems throughout this book will be based on data. Data-based homework problems require the use of a spreadsheet, graphing software, or a graphing calculator in which you can "fit" data with a straight line.

MODEL Model the cart as a particle.

VISUALIZE FIGURE 2.30 shows the pictorial representation. The track and axis are tilted at angle $\theta = \tan^{-1}(20.0\text{ cm}/180\text{ cm}) = 6.34°$. This is motion on an inclined plane, so you might expect the cart's acceleration to be $a_x = g\sin\theta = 1.08\text{ m/s}^2$.

FIGURE 2.30 The pictorial representation of the cart on the track.



Known
$x_0 = 0$ m $v_{0x} = 0$ m/s
$t_0 = 0$ s $\theta = 6.34°$

Find
a_x

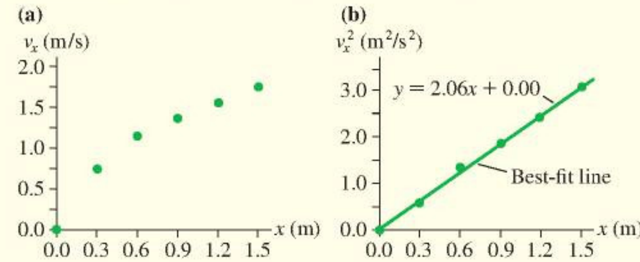
SOLVE In analyzing data, we want to use *all* the data. Further, we almost always want to use graphs when we have a series of measurements. We might start by graphing speed versus distance traveled. This is shown in FIGURE 2.31a, where we've converted distances to meters. As expected, speed increases with distance, but the graph isn't linear and that makes it hard to analyze.

Rather than proceeding by trial and error, let's be guided by theory. *If* the cart has constant acceleration—which we don't yet know and need to confirm—the third kinematic equation tells us that velocity and displacement should be related by

$$v_x^2 = v_{0x}^2 + 2a_x\Delta x = 2a_x x$$

The last step was based on starting from rest ($v_{0x} = 0$) at the origin ($\Delta x = x - x_0 = x$).

FIGURE 2.31 Graphs of velocity and of velocity squared.



Rather than graphing v_x versus x, suppose we graph v_x^2 versus x. If we let $y = v_x^2$, the kinematic equation reads

$$y = 2a_x x$$

This is in the form of a linear equation: $y = mx + b$, where m is the slope and b is the y-intercept. In this case, $m = 2a_x$ and $b = 0$. So if the cart really does have constant acceleration, a graph of v_x^2 versus x should be linear with a y-intercept of zero. This is a prediction that we can test.

Thus our analysis has three steps:

1. Graph v_x^2 versus x. If the graph is a straight line with a y-intercept of zero (or very close to zero), then we can conclude that the cart has constant acceleration on the ramp. If not, the acceleration is *not* constant and we cannot use the kinematic equations for constant acceleration.
2. If the graph has the correct shape, we can determine its slope m.
3. Because kinematics predicts $m = 2a_x$, the acceleration must be $a_x = m/2$.

FIGURE 2.31b is the graph of v_x^2 versus x. It does turn out to be a straight line with a y-intercept of zero, and this is the evidence we need that the cart has a constant acceleration on the ramp. To proceed, we want to determine the slope by finding the straight line that is the "best fit" to the data. This is a statistical technique, justified in a statistics class, but one that is implemented in spreadsheets and graphing calculators. The solid line in Figure 2.31b is the best-fit line for this data, and its equation is shown. We see that the slope is $m = 2.06\text{ m/s}^2$. **Slopes have units,** and the units come not from the fitting procedure but by looking at the axes of the graph. Here the vertical axis is velocity squared, with units of $(\text{m/s})^2$, while the horizontal axis is position, measured in m. Thus the slope, rise over run, has units of m/s^2.

Finally, we can determine that the cart's acceleration was

$$a_x = \frac{m}{2} = 1.03\text{ m/s}^2$$

This is about 5% less than the 1.08 m/s^2 we expected. Two possibilities come to mind. Perhaps the distances used to find the tilt angle weren't measured accurately. Or, more likely, the cart rolls with a small bit of friction. The predicted acceleration $a_x = g\sin\theta$ is for a *frictionless* inclined plane; any friction would decrease the acceleration.

ASSESS The acceleration is just slightly less than predicted for a frictionless incline, so the result is reasonable.

Thinking Graphically

A good way to solidify your intuitive understanding of motion is to consider the problem of a hard, smooth ball rolling on a smooth track. The track is made up of several straight segments connected together. Each segment may be either horizontal or inclined. Your task is to analyze the ball's motion graphically.

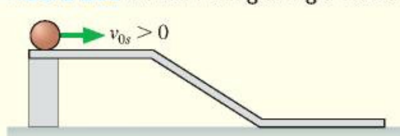
There are a small number of rules to follow:

1. Assume that the ball passes smoothly from one segment of the track to the next, with no abrupt change of speed and without ever leaving the track.
2. The graphs have no numbers, but they should show the correct *relationships*. For example, the position graph should be steeper in regions of higher speed.
3. The position s is the position measured *along* the track. Similarly, v_s and a_s are the velocity and acceleration parallel to the track.

EXAMPLE 2.15 From track to graphs

Draw position, velocity, and acceleration graphs for the ball on the smooth track of FIGURE 2.32.

FIGURE 2.32 A ball rolling along a track.

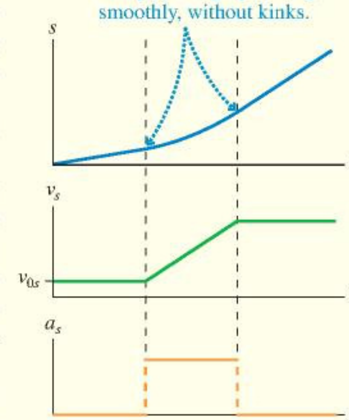


VISUALIZE It is often easiest to begin with the velocity. There is no acceleration on the horizontal surface ($a_s = 0$ if $\theta = 0°$), so the velocity remains constant at v_{0s} until the ball reaches the slope. The slope is an inclined plane where the ball has constant acceleration. The velocity increases linearly with time during constant-acceleration motion. The ball returns to constant-velocity motion after reaching the bottom horizontal segment. The middle graph of FIGURE 2.33 shows the velocity.

We can easily draw the acceleration graph. The acceleration is zero while the ball is on the horizontal segments and has a constant positive value on the slope. These accelerations are consistent with the slope of the velocity graph: zero slope, then positive slope, then a return to zero. The acceleration cannot *really* change instantly from zero to a nonzero value, but the change can be so quick that we do not see it on the time scale of the graph. That is what the vertical dotted lines imply.

Finally, we need to find the position-versus-time graph. The position increases linearly with time during the first segment at constant velocity. It also does so during the third segment of motion, but with a steeper slope to indicate a faster velocity. In between, while the acceleration is nonzero but constant, the position graph has a *parabolic* shape. Notice that the parabolic section blends *smoothly* into the straight lines on either side. An abrupt change of slope (a "kink") would indicate an abrupt change in velocity and would violate rule 1.

FIGURE 2.33 Motion graphs for the ball in Example 2.15.

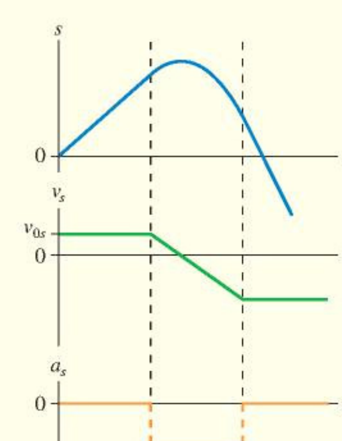


EXAMPLE 2.16 From graphs to track

FIGURE 2.34 shows a set of motion graphs for a ball moving on a track. Draw a picture of the track and describe the ball's initial condition. Each segment of the track is *straight*, but the segments may be tilted.

VISUALIZE The ball starts with initial velocity $v_{0s} > 0$ and maintains this velocity for awhile; there's no acceleration. Thus the ball must start out rolling to the right on a horizontal track. At the end of the motion, the ball is again rolling on a horizontal track (no acceleration, constant velocity), but it's rolling to the *left* because v_s is negative. Further, the final speed ($|v_s|$) is greater than the initial speed. The middle section of the graph shows us what happens. The ball starts slowing with constant acceleration (rolling uphill), reaches a turning point (s is maximum, $v_s = 0$), then speeds up in the opposite direction (rolling downhill). This is still a negative acceleration because the ball is speeding up in the negative

FIGURE 2.34 Motion graphs of a ball rolling on a track of unknown shape.

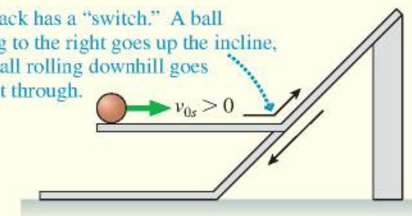


Continued

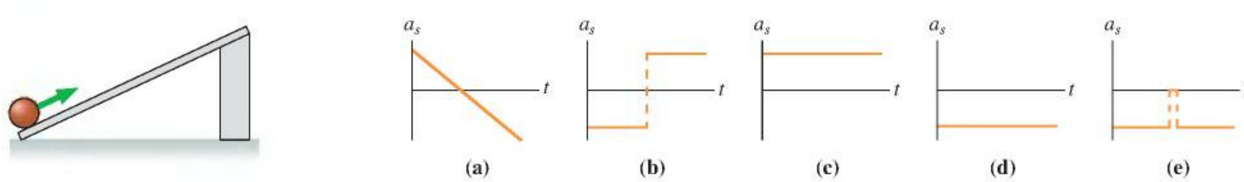
s-direction. It must roll farther downhill than it had rolled uphill before reaching a horizontal section of track. **FIGURE 2.35** shows the track and the initial conditions that are responsible for the graphs of Figure 2.34.

FIGURE 2.35 Track responsible for the motion graphs of Figure 2.34.

This track has a "switch." A ball moving to the right goes up the incline, but a ball rolling downhill goes straight through.

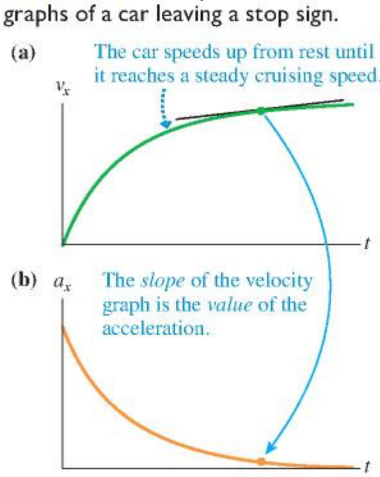


STOP TO THINK 2.5 The ball rolls up the ramp, then back down. Which is the correct acceleration graph?



2.7 ADVANCED TOPIC Instantaneous Acceleration

FIGURE 2.36 Velocity and acceleration graphs of a car leaving a stop sign.



Although the constant-acceleration model is very useful, real moving objects only rarely have constant acceleration. For example, **FIGURE 2.36a** is a realistic velocity-versus-time graph for a car leaving a stop sign. The graph is not a straight line, so this is *not* motion with constant acceleration.

We can define an instantaneous acceleration much as we defined the instantaneous velocity. The instantaneous velocity at time t is the slope of the position-versus-time graph at that time or, mathematically, the derivative of the position with respect to time. By analogy: **The instantaneous acceleration a_s is the slope of the line that is tangent to the velocity-versus-time curve at time t.** Mathematically, this is

$$a_s = \frac{dv_s}{dt} = \text{slope of the velocity-versus-time graph at time } t \qquad (2.27)$$

FIGURE 2.36b applies this idea by showing the car's acceleration graph. At each instant of time, the *value* of the car's acceleration is the *slope* of its velocity graph. The initially steep slope indicates a large initial acceleration. The acceleration decreases to zero as the car reaches cruising speed.

The reverse problem—to find the velocity v_s if we know the acceleration a_s at all instants of time—is also important. Again, with analogy to velocity and position, we have

$$v_{fs} = v_{is} + \int_{t_i}^{t_f} a_s\, dt \qquad (2.28)$$

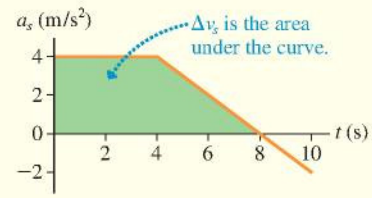
The graphical interpretation of Equation 2.28 is

$$v_{fs} = v_{is} + \text{area under the acceleration curve } a_s \text{ between } t_i \text{ and } t_f \qquad (2.29)$$

EXAMPLE 2.17 Finding velocity from acceleration

FIGURE 2.37 shows the acceleration graph for a particle with an initial velocity of 10 m/s. What is the particle's velocity at $t = 8$ s?

FIGURE 2.37 Acceleration graph for Example 2.17.



MODEL We're told this is the motion of a particle.

VISUALIZE Figure 2.37 is a graphical representation of the motion.

SOLVE The change in velocity is found as the area under the acceleration curve:

$$v_{fs} = v_{is} + \text{area under the acceleration curve } a_s \text{ between } t_i \text{ and } t_f$$

The area under the curve between $t_i = 0$ s and $t_f = 8$ s can be subdivided into a rectangle ($0 \text{ s} \le t \le 4$ s) and a triangle ($4 \text{ s} \le t \le 8$ s). These areas are easily computed. Thus

$$\begin{aligned} v_s(\text{at } t = 8 \text{ s}) &= 10 \text{ m/s} + (4 \text{ (m/s)/s})(4 \text{ s}) \\ &\quad + \tfrac{1}{2}(4 \text{ (m/s)/s})(4 \text{ s}) \\ &= 34 \text{ m/s} \end{aligned}$$

EXAMPLE 2.18 A realistic car acceleration

Starting from rest, a car takes T seconds to reach its cruising speed v_{max}. A plausible expression for the velocity as a function of time is

$$v_x(t) = \begin{cases} v_{max}\left(\dfrac{2t}{T} - \dfrac{t^2}{T^2}\right) & t \le T \\ v_{max} & t \ge T \end{cases}$$

a. Demonstrate that this is a plausible function by drawing velocity and acceleration graphs.

b. Find an expression for the distance traveled at time T in terms of T and the maximum acceleration a_{max}.

c. What are the maximum acceleration and the distance traveled for a car that reaches a cruising speed of 15 m/s in 8.0 s?

MODEL Model the car as a particle.

VISUALIZE **FIGURE 2.38a** shows the velocity graph. It's an inverted parabola that reaches v_{max} at time T and then holds that value. From the slope, we see that the acceleration should start at a maximum value a_{max}, steadily decrease until T, and be zero for $t > T$.

SOLVE

a. We can find an expression for a_x by taking the derivative of v_x. Starting with $t \le T$, and using Equation 2.6 for the derivatives of polynomials, we find

$$a_x = \frac{dv_x}{dt} = v_{max}\left(\frac{2}{T} - \frac{2t}{T^2}\right) = \frac{2v_{max}}{T}\left(1 - \frac{t}{T}\right) = a_{max}\left(1 - \frac{t}{T}\right)$$

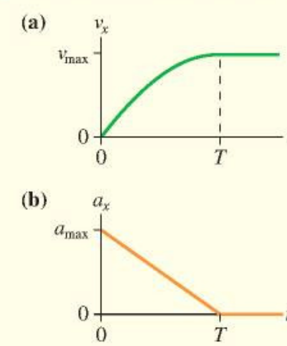
where $a_{max} = 2v_{max}/T$. For $t \ge T$, $a_x = 0$. Altogether,

$$a_x(t) = \begin{cases} a_{max}\left(1 - \dfrac{t}{T}\right) & t \le T \\ 0 & t \ge T \end{cases}$$

This expression for the acceleration is graphed in **FIGURE 2.38b**. The acceleration decreases linearly from a_{max} to 0 as the car accelerates from rest to its cruising speed.

b. To find the position as a function of time, we need to integrate the velocity (Equation 2.11) using Equation 2.13 for the integrals of polynomials. At time T, when cruising speed is reached,

FIGURE 2.38 Velocity and acceleration graphs for Example 2.18.



$$\begin{aligned} x_T = x_0 + \int_0^T v_x\,dt &= 0 + \frac{2v_{max}}{T}\int_0^T t\,dt - \frac{v_{max}}{T^2}\int_0^T t^2\,dt \\ &= \frac{2v_{max}}{T}\frac{t^2}{2}\bigg|_0^T - \frac{v_{max}}{T^2}\frac{t^3}{3}\bigg|_0^T \\ &= v_{max}T - \tfrac{1}{3}v_{max}T = \tfrac{2}{3}v_{max}T \end{aligned}$$

Recalling that $a_{max} = 2v_{max}/T$, we can write the distance traveled as

$$x_T = \tfrac{2}{3}v_{max}T = \tfrac{1}{3}\left(\frac{2v_{max}}{T}\right)T^2 = \tfrac{1}{3}a_{max}T^2$$

If the acceleration stayed constant, the distance would be $\frac{1}{2}aT^2$. We have found a similar expression but, because the acceleration is steadily decreasing, a smaller fraction in front.

c. With $v_{max} = 15$ m/s and $T = 8.0$ s, realistic values for city driving, we find

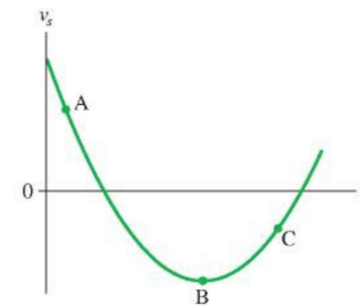
$$a_{max} = \frac{2v_{max}}{T} = \frac{2(15 \text{ m/s})}{8.0 \text{ s}} = 3.75 \text{ m/s}^2$$

$$x_T = \tfrac{1}{3}a_{max}T^2 = \tfrac{1}{3}(3.75 \text{ m/s}^2)(8.0 \text{ s})^2 = 80 \text{ m}$$

ASSESS 80 m in 8.0 s to reach a cruising speed of 15 m/s ≈ 30 mph is very reasonable. This gives us good reason to believe that a car's initial acceleration is $\approx \frac{1}{3}g$.

STOP TO THINK 2.6 Rank in order, from most positive to least positive, the accelerations at points A to C.

a. $a_A > a_B > a_C$
b. $a_C > a_A > a_B$
c. $a_C > a_B > a_A$
d. $a_B > a_A > a_C$



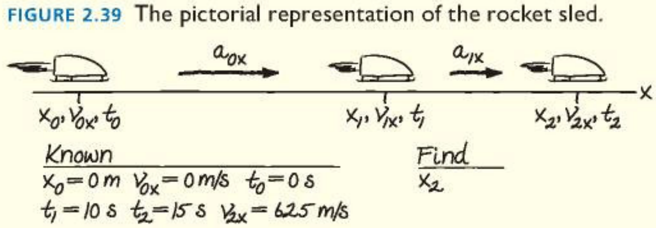
CHALLENGE EXAMPLE 2.19 | Rocketing along

A rocket sled accelerates along a long, horizontal rail. Starting from rest, two rockets burn for 10 s, providing a constant acceleration. One rocket then burns out, halving the acceleration, but the other burns for an additional 5 s to boost the sled's speed to 625 m/s. How far has the sled traveled when the second rocket burns out?

MODEL Model the rocket sled as a particle with constant acceleration.

VISUALIZE FIGURE 2.39 shows the pictorial representation. This is a two-part problem with a beginning, an end (the second rocket burns out), and a point in between where the motion changes (the first rocket burns out).

FIGURE 2.39 The pictorial representation of the rocket sled.



SOLVE The difficulty with this problem is that there's not enough information to completely analyze either the first or the second part of the motion. A successful solution will require combining information about both parts of the motion, and that can be done only by working algebraically, not worrying about numbers until the end of the problem. A well-drawn pictorial representation and clearly defined symbols are essential.

The first part of the motion, with both rockets firing, has acceleration a_{0x}. The sled's position and velocity when the first rocket burns out are

$$x_1 = x_0 + v_{0x}\,\Delta t + \tfrac{1}{2}a_{0x}(\Delta t)^2 = \tfrac{1}{2}a_{0x}t_1^{\,2}$$
$$v_{1x} = v_{0x} + a_{0x}\,\Delta t = a_{0x}t_1$$

where we simplified as much as possible by knowing that the sled started from rest at the origin at $t_0 = 0$ s. We can't compute numerical values, but these are valid algebraic expressions that we can carry over to the second part of the motion.

From t_1 to t_2, the acceleration is a smaller a_{1x}. The velocity when the second rocket burns out is

$$v_{2x} = v_{1x} + a_{1x}\,\Delta t = a_{0x}t_1 + a_{1x}(t_2 - t_1)$$

where for v_{1x} we used the algebraic result from the first part of the motion. Now we have enough information to complete the solution. We know that the acceleration is halved when the first rocket burns out, so $a_{1x} = \frac{1}{2}a_{0x}$. Thus

$$v_{2x} = 625\text{ m/s} = a_{0x}(10\text{ s}) + \tfrac{1}{2}a_{0x}(5\text{ s}) = (12.5\text{ s})a_{0x}$$

Solving, we find $a_{0x} = 50\text{ m/s}^2$.

With the acceleration now known, we can calculate the position and velocity when the first rocket burns out:

$$x_1 = \tfrac{1}{2}a_{0x}t_1^{\,2} = \tfrac{1}{2}(50\text{ m/s}^2)(10\text{ s})^2 = 2500\text{ m}$$
$$v_{1x} = a_{0x}t_1 = (50\text{ m/s}^2)(10\text{ s}) = 500\text{ m/s}$$

Finally, the position when the second rocket burns out is

$$x_2 = x_1 + v_{1x}\,\Delta t + \tfrac{1}{2}a_{1x}(\Delta t)^2$$
$$= 2500\text{ m} + (500\text{ m/s})(5\text{ s}) + \tfrac{1}{2}(25\text{ m/s}^2)(5\text{ s})^2 = 5300\text{ m}$$

The sled has traveled 5300 m when it reaches 625 m/s at the burnout of the second rocket.

ASSESS 5300 m is 5.3 km, or roughly 3 miles. That's a long way to travel in 15 s! But the sled reaches incredibly high speeds. At the final speed of 625 m/s, over 1200 mph, the sled would travel nearly 10 km in 15 s. So 5.3 km in 15 s for the accelerating sled seems reasonable.

SUMMARY

The goal of Chapter 2 has been to learn to solve problems about motion along a straight line.

GENERAL PRINCIPLES

Kinematics describes motion in terms of position, velocity, and acceleration. General kinematic relationships are given **mathematically** by:

Instantaneous velocity $v_s = ds/dt =$ slope of position graph

Instantaneous acceleration $a_s = dv_s/dt =$ slope of velocity graph

Final position $s_f = s_i + \int_{t_i}^{t_f} v_s\,dt = s_i + \begin{cases}\text{area under the velocity}\\ \text{curve from } t_i \text{ to } t_f\end{cases}$

Final velocity $v_{fs} = v_{is} + \int_{t_i}^{t_f} a_s\,dt = v_{is} + \begin{cases}\text{area under the acceleration}\\ \text{curve from } t_i \text{ to } t_f\end{cases}$

Solving Kinematics Problems

MODEL Uniform motion or constant acceleration.

VISUALIZE Draw a pictorial representation.

SOLVE

- Uniform motion $s_f = s_i + v_s \Delta t$
- Constant acceleration $v_{fs} = v_{is} + a_s \Delta t$

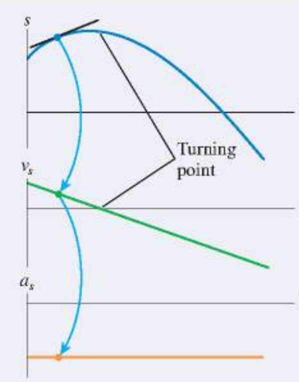
$$s_f = s_i + v_s \Delta t + \tfrac{1}{2} a_s (\Delta t)^2$$
$$v_{fs}^{\,2} = v_{is}^{\,2} + 2 a_s \Delta s$$

ASSESS Is the result reasonable?

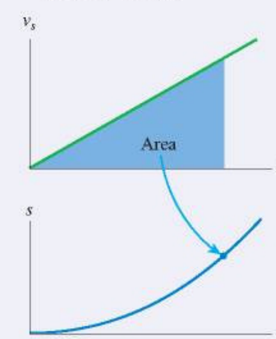
IMPORTANT CONCEPTS

Position, velocity, and acceleration are related graphically.

- The slope of the position-versus-time graph is the value on the velocity graph.
- The slope of the velocity graph is the value on the acceleration graph.
- s is a maximum or minimum at a turning point, and $v_s = 0$.



- Displacement is the area under the velocity curve.



APPLICATIONS

The **sign of v_s** indicates the direction of motion.

- $v_s > 0$ is motion to the right or up.
- $v_s < 0$ is motion to the left or down.

The **sign of a_s** indicates which way $\vec{a}$ points, *not* whether the object is speeding up or slowing down.

- $a_s > 0$ if $\vec{a}$ points to the right or up.
- $a_s < 0$ if $\vec{a}$ points to the left or down.
- The direction of $\vec{a}$ is found with a motion diagram.

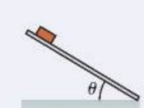
An object is **speeding up** if and only if v_s and a_s have the same sign.

An object is **slowing down** if and only if v_s and a_s have opposite signs.

Free fall is constant-acceleration motion with

$$a_y = -g = -9.80 \text{ m/s}^2$$

Motion on an inclined plane has $a_s = \pm g \sin\theta$. The sign depends on the direction of the tilt.



TERMS AND NOTATION

kinematics
uniform motion
average velocity, v_{avg}
speed, v
initial position, s_i
final position, s_f
uniform-motion model
instantaneous velocity, v_s
turning point
average acceleration, a_{avg}
constant-acceleration model
free fall
free-fall acceleration, g
instantaneous acceleration, a_s

CONCEPTUAL QUESTIONS

For Questions 1 through 3, interpret the position graph given in each figure by writing a very short "story" of what is happening. Be creative! Have characters and situations! Simply saying that "a car moves 100 meters to the right" doesn't qualify as a story. Your stories should make *specific reference* to information you obtain from the graph, such as distance moved or time elapsed.

1.

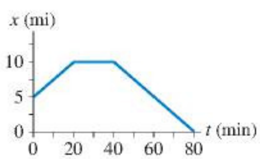


FIGURE Q2.1

2.

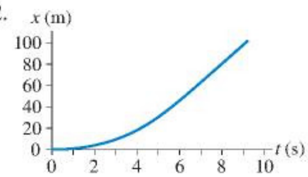


FIGURE Q2.2

3.

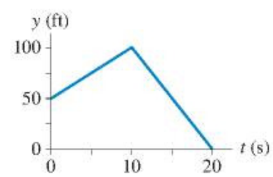


FIGURE Q2.3

4. FIGURE Q2.4 shows a position-versus-time graph for the motion of objects A and B as they move along the same axis.
   a. At the instant $t = 1$ s, is the speed of A greater than, less than, or equal to the speed of B? Explain.
   b. Do objects A and B ever have the *same* speed? If so, at what time or times? Explain.

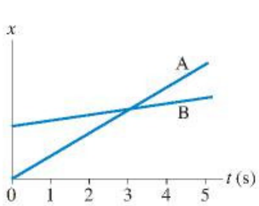


FIGURE Q2.4

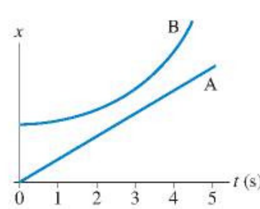


FIGURE Q2.5

5. FIGURE Q2.5 shows a position-versus-time graph for the motion of objects A and B as they move along the same axis.
   a. At the instant $t = 1$ s, is the speed of A greater than, less than, or equal to the speed of B? Explain.
   b. Do objects A and B ever have the *same* speed? If so, at what time or times? Explain.
6. FIGURE Q2.6 shows the position-versus-time graph for a moving object. At which lettered point or points:
   a. Is the object *moving* the slowest?
   b. Is the object moving the fastest?
   c. Is the object at rest?
   d. Is the object moving to the left?

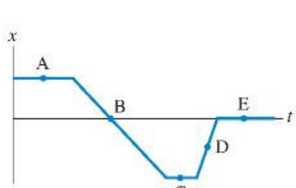


FIGURE Q2.6

7. FIGURE Q2.7 shows the position-versus-time graph for a moving object. At which lettered point or points:
   a. Is the object moving the fastest?
   b. Is the object moving to the left?
   c. Is the object speeding up?
   d. Is the object turning around?

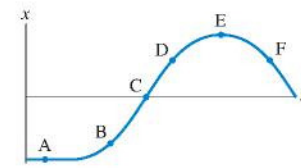


FIGURE Q2.7

8. FIGURE Q2.8 shows six frames from the motion diagrams of two moving cars, A and B.
   a. Do the two cars ever have the same position at one instant of time? If so, in which frame number (or numbers)?
   b. Do the two cars ever have the same velocity at one instant of time? If so, between which two frames?

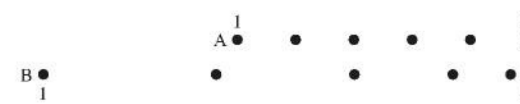


FIGURE Q2.8

9. You're driving along the highway at a steady speed of 60 mph when another driver decides to pass you. At the moment when the front of his car is exactly even with the front of your car, and you turn your head to smile at him, do the two cars have equal velocities? Explain.
10. A bicycle is traveling east. Can its acceleration vector ever point west? Explain.
11. (a) Give an example of a vertical motion with a positive velocity and a negative acceleration. (b) Give an example of a vertical motion with a negative velocity and a negative acceleration.
12. A ball is thrown straight up into the air. At each of the following instants, is the magnitude of the ball's acceleration greater than g, equal to g, less than g, or 0? Explain.
    a. Just after leaving your hand.
    b. At the very top (maximum height).
    c. Just before hitting the ground.
13. A rock is *thrown* (not dropped) straight down from a bridge into the river below. At each of the following instants, is the magnitude of the rock's acceleration greater than g, equal to g, less than g, or 0? Explain.
    a. Immediately after being released.
    b. Just before hitting the water.
14. FIGURE Q2.14 shows the velocity-versus-time graph for a moving object. At which lettered point or points:
    a. Is the object speeding up?
    b. Is the object slowing down?
    c. Is the object moving to the left?
    d. Is the object moving to the right?

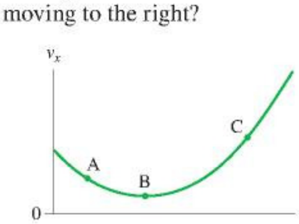


FIGURE Q2.14

EXERCISES AND PROBLEMS

Exercises

Section 2.1 Uniform Motion

1. | Alan leaves Los Angeles at 8:00 A.M. to drive to San Francisco, 400 mi away. He travels at a steady 50 mph. Beth leaves Los Angeles at 9:00 A.M. and drives a steady 60 mph.
   a. Who gets to San Francisco first?
   b. How long does the first to arrive have to wait for the second?
2. | Julie drives 100 mi to Grandmother's house. On the way to Grandmother's, Julie drives half the distance at 40 mph and half the distance at 60 mph. On her return trip, she drives half the time at 40 mph and half the time at 60 mph.
   a. What is Julie's average speed on the way to Grandmother's house?
   b. What is her average speed on the return trip?
3. | Larry leaves home at 9:05 and runs at constant speed to the lamppost seen in FIGURE EX2.3. He reaches the lamppost at 9:07, immediately turns, and runs to the tree. Larry arrives at the tree at 9:10.
   a. What is Larry's average velocity, in m/min, during each of these two intervals?
   b. What is Larry's average velocity for the entire run?

FIGURE EX2.3

4. | FIGURE EX2.4 is the position-versus-time graph of a jogger. What is the jogger's velocity at $t = 10$ s, at $t = 25$ s, and at $t = 35$ s?

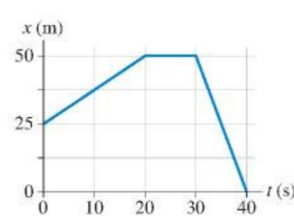


FIGURE EX2.4

Section 2.2 Instantaneous Velocity

Section 2.3 Finding Position from Velocity

5. | FIGURE EX2.5 shows the position graph of a particle.
   a. Draw the particle's velocity graph for the interval $0 \text{ s} \le t \le 4$ s.
   b. Does this particle have a turning point or points? If so, at what time or times?

FIGURE EX2.5

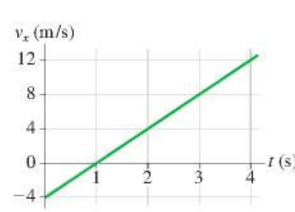


FIGURE EX2.6

6. || A particle starts from $x_0 = 10$ m at $t_0 = 0$ s and moves with the velocity graph shown in FIGURE EX2.6.
   a. Does this particle have a turning point? If so, at what time?
   b. What is the object's position at $t = 2$ s and 4 s?

7. | FIGURE EX2.7 is a somewhat idealized graph of the velocity of blood in the ascending aorta during one beat of the heart. Approximately how far, in cm, does the blood move during one beat?
BIO

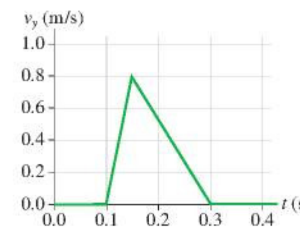


FIGURE EX2.7

8. | FIGURE EX2.8 shows the velocity graph for a particle having initial position $x_0 = 0$ m at $t_0 = 0$ s. At what time or times is the particle found at $x = 35$ m?

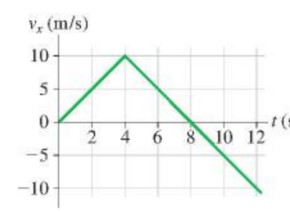


FIGURE EX2.8

Section 2.4 Motion with Constant Acceleration

9. | FIGURE EX2.9 shows the velocity graph of a particle. Draw the particle's acceleration graph for the interval $0 \text{ s} \le t \le 4$ s.

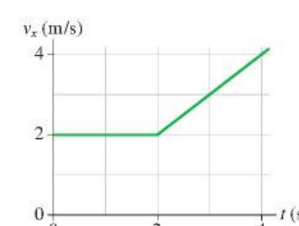


FIGURE EX2.9

10. | FIGURE EX2.7 showed the velocity graph of blood in the aorta. What is the blood's acceleration during each phase of the motion, speeding up and slowing down?
BIO
11. | FIGURE EX2.11 shows the velocity graph of a particle moving along the x-axis. Its initial position is $x_0 = 2.0$ m at $t_0 = 0$ s. At $t = 2.0$ s, what are the particle's (a) position, (b) velocity, and (c) acceleration?

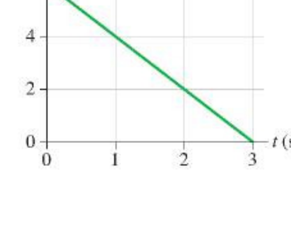


FIGURE EX2.11

12. || FIGURE EX2.12 shows the velocity-versus-time graph for a particle moving along the x-axis. Its initial position is $x_0 = 2.0$ m at $t_0 = 0$ s.
 a. What are the particle's position, velocity, and acceleration at $t = 1.0$ s?
 b. What are the particle's position, velocity, and acceleration at $t = 3.0$ s?

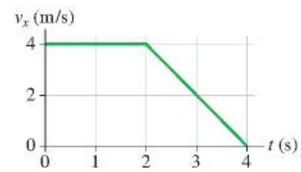


FIGURE EX2.12

13. | a. What constant acceleration, in SI units, must a car have to go from zero to 60 mph in 10 s?
 b. How far has the car traveled when it reaches 60 mph? Give your answer both in SI units and in feet.

14. || A jet plane is cruising at 300 m/s when suddenly the pilot turns the engines up to full throttle. After traveling 4.0 km, the jet is moving with a speed of 400 m/s. What is the jet's acceleration, assuming it to be a constant acceleration?

15. || a. How many days will it take a spaceship to accelerate to the speed of light (3.0×10^8 m/s) with the acceleration g?
 b. How far will it travel during this interval?
 c. What fraction of a light year is your answer to part b? A light year is the distance light travels in one year.

NOTE We know, from Einstein's theory of relativity, that no object can travel at the speed of light. So this problem, while interesting and instructive, is not realistic.

16. || When you sneeze, the air in your lungs accelerates from rest to 150 km/h in approximately 0.50 s. What is the acceleration of the air in m/s^2?
BIO

17. || A speed skater moving to the left across frictionless ice at 8.0 m/s hits a 5.0-m-wide patch of rough ice. She slows steadily, then continues on at 6.0 m/s. What is her acceleration on the rough ice?

18. || A Porsche challenges a Honda to a 400 m race. Because the Porsche's acceleration of 3.5 m/s^2 is larger than the Honda's 3.0 m/s^2, the Honda gets a 1.0 s head start. Who wins? By how many seconds?

19. || A car starts from rest at a stop sign. It accelerates at 4.0 m/s^2 for 6.0 s, coasts for 2.0 s, and then slows down at a rate of 3.0 m/s^2 for the next stop sign. How far apart are the stop signs?

Section 2.5 Free Fall

20. | Ball bearings are made by letting spherical drops of molten metal fall inside a tall tower—called a *shot tower*—and solidify as they fall.
 a. If a bearing needs 4.0 s to solidify enough for impact, how high must the tower be?
 b. What is the bearing's impact velocity?

21. || A student standing on the ground throws a ball straight up. The ball leaves the student's hand with a speed of 15 m/s when the hand is 2.0 m above the ground. How long is the ball in the air before it hits the ground? (The student moves her hand out of the way.)

22. || A rock is tossed straight up from ground level with a speed of 20 m/s. When it returns, it falls into a hole 10 m deep.
 a. What is the rock's velocity as it hits the bottom of the hole?
 b. How long is the rock in the air, from the instant it is released until it hits the bottom of the hole?

23. || When jumping, a flea accelerates at an astounding 1000 m/s^2, but over only the very short distance of 0.50 mm. If a flea jumps straight up, and if air resistance is neglected (a rather poor approximation in this situation), how high does the flea go?
BIO

24. || As a science project, you drop a watermelon off the top of the Empire State Building, 320 m above the sidewalk. It so happens that Superman flies by at the instant you release the watermelon. Superman is headed straight down with a speed of 35 m/s. How fast is the watermelon going when it passes Superman?

25. ||| A rock is dropped from the top of a tall building. The rock's displacement in the last second before it hits the ground is 45% of the entire distance it falls. How tall is the building?

Section 2.6 Motion on an Inclined Plane

26. || A skier is gliding along at 3.0 m/s on horizontal, frictionless snow. He suddenly starts down a 10° incline. His speed at the bottom is 15 m/s.
 a. What is the length of the incline?
 b. How long does it take him to reach the bottom?

27. || A car traveling at 30 m/s runs out of gas while traveling up a 10° slope. How far up the hill will it coast before starting to roll back down?

28. || Santa loses his footing and slides down a frictionless, snowy roof that is tilted at an angle of 30°. If Santa slides 10 m before reaching the edge, what is his speed as he leaves the roof?

29. || A snowboarder glides down a 50-m-long, 15° hill. She then glides horizontally for 10 m before reaching a 25° upward slope. Assume the snow is frictionless.
 a. What is her velocity at the bottom of the hill?
 b. How far can she travel up the 25° slope?

30. || A small child gives a plastic frog a big push at the bottom of a slippery 2.0-m-long, 1.0-m-high ramp, starting it with a speed of 5.0 m/s. What is the frog's speed as it flies off the top of the ramp?

Section 2.7 Instantaneous Acceleration

31. || FIGURE EX2.31 shows the acceleration-versus-time graph of a particle moving along the x-axis. Its initial velocity is $v_{0x} = 8.0$ m/s at $t_0 = 0$ s. What is the particle's velocity at $t = 4.0$ s?

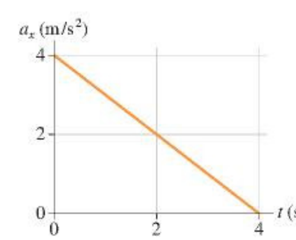


FIGURE EX2.31

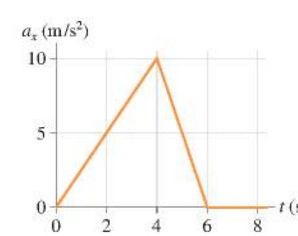


FIGURE EX2.32

32. || FIGURE EX2.32 shows the acceleration graph for a particle that starts from rest at $t = 0$ s. What is the particle's velocity at $t = 6$ s?

33. | A particle moving along the x-axis has its position described by the function $x = (2t^3 + 2t + 1)$ m, where t is in s. At $t = 2$ s what are the particle's (a) position, (b) velocity, and (c) acceleration?
CALC

34. || A particle moving along the x-axis has its velocity described
CALC by the function $v_x = 2t^2$ m/s, where t is in s. Its initial position is $x_0 = 1$ m at $t_0 = 0$ s. At $t = 1$ s what are the particle's (a) position, (b) velocity, and (c) acceleration?

35. | The position of a particle is given by the function
CALC $x = (2t^3 - 9t^2 + 12)$ m, where t is in s.
   a. At what time or times is $v_x = 0$ m/s?
   b. What are the particle's position and its acceleration at this time(s)?

36. || The position of a particle is given by the function
CALC $x = (2t^3 - 6t^2 + 12)$ m, where t is in s.
   a. At what time does the particle reach its minimum velocity? What is $(v_x)_{min}$?
   b. At what time is the acceleration zero?

Problems

37. || Particles A, B, and C move along the x-axis. Particle C has an initial velocity of 10 m/s. In FIGURE P2.37, the graph for A is a position-versus-time graph; the graph for B is a velocity-versus-time graph; the graph for C is an acceleration-versus-time graph. Find each particle's velocity at $t = 7.0$ s.

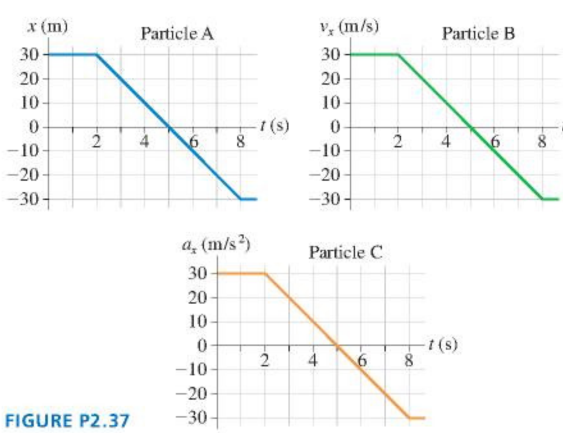


FIGURE P2.37

38. | A block is suspended from a spring, pulled down, and released. The block's position-versus-time graph is shown in FIGURE P2.38.
   a. At what times is the velocity zero? At what times is the velocity most positive? Most negative?
   b. Draw a reasonable velocity-versus-time graph.

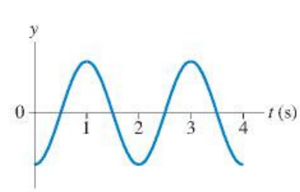


FIGURE P2.38

39. || A particle's velocity is described by the function
CALC $v_x = (t^2 - 7t + 10)$ m/s, where t is in s.
   a. At what times does the particle reach its turning points?
   b. What is the particle's acceleration at each of the turning points?

40. ||| A particle's velocity is described by the function $v_x = kt^2$ m/s,
CALC where k is a constant and t is in s. The particle's position at $t_0 = 0$ s is $x_0 = -9.0$ m. At $t_1 = 3.0$ s, the particle is at $x_1 = 9.0$ m. Determine the value of the constant k. Be sure to include the proper units.

41. || A particle's acceleration is described by the function
CALC $a_x = (10 - t)$ m/s², where t is in s. Its initial conditions are $x_0 = 0$ m and $v_{0x} = 0$ m/s at $t = 0$ s.
   a. At what time is the velocity again zero?
   b. What is the particle's position at that time?

42. || A particle's velocity is given by the function
CALC $v_x = (2.0 \text{ m/s})\sin(\pi t)$, where t is in s.
   a. What is the first time after $t = 0$ s when the particle reaches a turning point?
   b. What is the particle's acceleration at that time?

43. || A ball rolls along the smooth track shown in FIGURE P2.43. Each segment of the track is straight, and the ball passes smoothly from one segment to the next without changing speed or leaving the track. Draw three vertically stacked graphs showing position, velocity, and acceleration versus time. Each graph should have the same time axis, and the proportions of the graph should be qualitatively correct. Assume that the ball has enough speed to reach the top.

FIGURE P2.43

44. || Draw position, velocity, and acceleration graphs for the ball shown in FIGURE P2.44. See Problem 43 for more information.

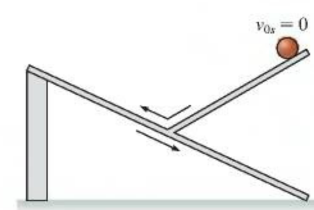


FIGURE P2.44

45. || FIGURE P2.45 shows a set of kinematic graphs for a ball rolling on a track. All segments of the track are straight lines, but some may be tilted. Draw a picture of the track and also indicate the ball's initial condition.

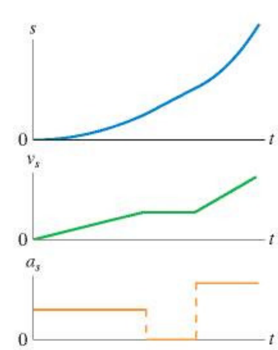


FIGURE P2.45

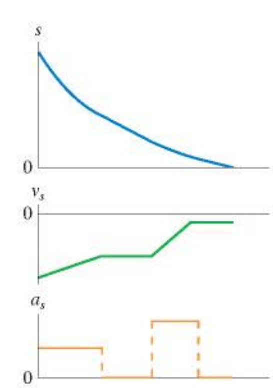


FIGURE P2.46

46. || FIGURE P2.46 shows a set of kinematic graphs for a ball rolling on a track. All segments of the track are straight lines, but some may be tilted. Draw a picture of the track and also indicate the ball's initial condition.

47. ‖ The takeoff speed for an Airbus A320 jetliner is 80 m/s. Velocity data measured during takeoff are as shown.

t (s)	v_x (m/s)
0	0
10	23
20	46
30	69

a. Is the jetliner's acceleration constant during takeoff? Explain.
b. At what time do the wheels leave the ground?
c. For safety reasons, in case of an aborted takeoff, the runway must be three times the takeoff distance. Can an A320 take off safely on a 2.5-mi-long runway?

48. | You are driving to the grocery store at 20 m/s. You are 110 m from an intersection when the traffic light turns red. Assume that your reaction time is 0.50 s and that your car brakes with constant acceleration. What magnitude braking acceleration will bring you to a stop exactly at the intersection?

49. ‖ You're driving down the highway late one night at 20 m/s when a deer steps onto the road 35 m in front of you. Your reaction time before stepping on the brakes is 0.50 s, and the maximum deceleration of your car is 10 m/s^2.
a. How much distance is between you and the deer when you come to a stop?
b. What is the maximum speed you could have and still not hit the deer?

50. ‖ Two cars are driving at the same constant speed on a straight road, with car 1 in front of car 2. Car 1 suddenly starts to brake with constant acceleration and stops in 10 m. At the instant car 1 comes to a stop, car 2 begins to brake with the same acceleration. It comes to a halt just as it reaches the back of car 1. What was the separation between the cars before they starting braking?

51. ‖ You are playing miniature golf at the golf course shown in FIGURE P2.51. Due to the fake plastic grass, the ball decelerates at 1.0 m/s^2 when rolling horizontally and at 6.0 m/s^2 on the slope. What is the slowest speed with which the ball can leave your golf club if you wish to make a hole in one?

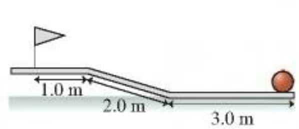


FIGURE P2.51

52. ‖ The minimum stopping distance for a car traveling at a speed of 30 m/s is 60 m, including the distance traveled during the driver's reaction time of 0.50 s. What is the minimum stopping distance for the same car traveling at a speed of 40 m/s?

53. ‖ A cheetah spots a Thomson's gazelle, its preferred prey, and
BIO leaps into action, quickly accelerating to its top speed of 30 m/s, the highest of any land animal. However, a cheetah can maintain this extreme speed for only 15 s before having to let up. The cheetah is 170 m from the gazelle as it reaches top speed, and the gazelle sees the cheetah at just this instant. With negligible reaction time, the gazelle heads directly away from the cheetah, accelerating at 4.6 m/s^2 for 5.0 s, then running at constant speed. Does the gazelle escape? If so, by what distance is the gazelle in front when the cheetah gives up?

54. ‖ You are at a train station, standing next to the train at the front of the first car. The train starts moving with constant acceleration, and 5.0 s later the back of the first car passes you. How long does it take after the train starts moving until the back of the seventh car passes you? All cars are the same length.

55. ||| A 200 kg weather rocket is loaded with 100 kg of fuel and fired straight up. It accelerates upward at 30 m/s^2 for 30 s, then runs out of fuel. Ignore any air resistance effects.
a. What is the rocket's maximum altitude?
b. How long is the rocket in the air before hitting the ground?

56. ||| A 1000 kg weather rocket is launched straight up. The rocket motor provides a constant acceleration for 16 s, then the motor stops. The rocket altitude 20 s after launch is 5100 m. You can ignore any effects of air resistance. What was the rocket's acceleration during the first 16 s?

57. ||| A lead ball is dropped into a lake from a diving board 5.0 m above the water. After entering the water, it sinks to the bottom with a constant velocity equal to the velocity with which it hit the water. The ball reaches the bottom 3.0 s after it is released. How deep is the lake?

58. ‖ A hotel elevator ascends 200 m with a maximum speed of 5.0 m/s. Its acceleration and deceleration both have a magnitude of 1.0 m/s^2.
a. How far does the elevator move while accelerating to full speed from rest?
b. How long does it take to make the complete trip from bottom to top?

59. ‖ A basketball player can jump to a height of 55 cm. How far above the floor can he jump in an elevator that is descending at a constant 1.0 m/s?

60. ‖ You are 9.0 m from the door of your bus, behind the bus, when it pulls away with an acceleration of 1.0 m/s^2. You instantly start running toward the still-open door at 4.5 m/s.
a. How long does it take for you to reach the open door and jump in?
b. What is the maximum time you can wait before starting to run and still catch the bus?

61. ‖ Ann and Carol are driving their cars along the same straight road. Carol is located at $x = 2.4$ mi at $t = 0$ h and drives at a steady 36 mph. Ann, who is traveling in the same direction, is located at $x = 0.0$ mi at $t = 0.50$ h and drives at a steady 50 mph.
a. At what time does Ann overtake Carol?
b. What is their position at this instant?
c. Draw a position-versus-time graph showing the motion of both Ann and Carol.

62. ‖ Amir starts riding his bike up a 200-m-long slope at a speed of 18 km/h, decelerating at 0.20 m/s^2 as he goes up. At the same instant, Becky starts down from the top at a speed of 6.0 km/h, accelerating at 0.40 m/s^2 as she goes down. How far has Amir ridden when they pass?

63. ‖ A very slippery block of ice slides down a smooth ramp tilted at angle θ. The ice is released from rest at vertical height h above the bottom of the ramp. Find an expression for the speed of the ice at the bottom.

64. ‖ Bob is driving the getaway car after the big bank robbery. He's going 50 m/s when his headlights suddenly reveal a nail strip that the cops have placed across the road 150 m in front of him. If Bob can stop in time, he can throw the car into reverse and escape. But if he crosses the nail strip, all his tires will go flat and he will be caught. Bob's reaction time before he can hit the brakes is 0.60 s, and his car's maximum deceleration is 10 m/s^2. Does Bob stop before or after the nail strip? By what distance?

65. || One game at the amusement park has you push a puck up a long, frictionless ramp. You win a stuffed animal if the puck, at its highest point, comes to within 10 cm of the end of the ramp without going off. You give the puck a push, releasing it with a speed of 5.0 m/s when it is 8.5 m from the end of the ramp. The puck's speed after traveling 3.0 m is 4.0 m/s. How far is it from the end when it stops?
66. || A motorist is driving at 20 m/s when she sees that a traffic light 200 m ahead has just turned red. She knows that this light stays red for 15 s, and she wants to reach the light just as it turns green again. It takes her 1.0 s to step on the brakes and begin slowing. What is her speed as she reaches the light at the instant it turns green?
67. || Nicole throws a ball straight up. Chad watches the ball from a window 5.0 m above the point where Nicole released it. The ball passes Chad on the way up, and it has a speed of 10 m/s as it passes him on the way back down. How fast did Nicole throw the ball?
68. || David is driving a steady 30 m/s when he passes Tina, who is sitting in her car at rest. Tina begins to accelerate at a steady 2.0 m/s^2 at the instant when David passes.
    a. How far does Tina drive before passing David?
    b. What is her speed as she passes him?
69. || A cat is sleeping on the floor in the middle of a 3.0-m-wide room when a barking dog enters with a speed of 1.50 m/s. As the dog enters, the cat (as only cats can do) immediately accelerates at 0.85 m/s^2 toward an open window on the opposite side of the room. The dog (all bark and no bite) is a bit startled by the cat and begins to slow down at 0.10 m/s^2 as soon as it enters the room. How far is the cat in front of the dog as it leaps through the window?
70. ||| Water drops fall from the edge of a roof at a steady rate. A fifth drop starts to fall just as the first drop hits the ground. At this instant, the second and third drops are exactly at the bottom and top edges of a 1.00-m-tall window. How high is the edge of the roof?
71. ||| I was driving along at 20 m/s, trying to change a CD and not watching where I was going. When I looked up, I found myself 45 m from a railroad crossing. And wouldn't you know it, a train moving at 30 m/s was only 60 m from the crossing. In a split second, I realized that the train was going to beat me to the crossing and that I didn't have enough distance to stop. My only hope was to accelerate enough to cross the tracks before the train arrived. If my reaction time before starting to accelerate was 0.50 s, what minimum acceleration did my car need for me to be here today writing these words?
72. || As an astronaut visiting Planet X, you're assigned to measure the free-fall acceleration. Getting out your meter stick and stop watch, you time the fall of a heavy ball from several heights. Your data are as follows:

Height (m)	Fall time (s)
0.0	0.00
1.0	0.54
2.0	0.72
3.0	0.91
4.0	1.01
5.0	1.17

Analyze these data to determine the free-fall acceleration on Planet X. Your analysis method should involve fitting a straight line to an appropriate graph, similar to the analysis in Example 2.14.

73. || Your goal in laboratory is to launch a ball of mass m straight up so that it reaches exactly height h above the top of the launching tube. You and your lab partners will earn fewer points if the ball goes too high or too low. The launch tube uses compressed air to accelerate the ball over a distance d, and you have a table of data telling you how to set the air compressor to achieve a desired acceleration. Find an expression for the acceleration that will earn you maximum points.
74. || CALC When a 1984 Alfa Romeo Spider sports car accelerates at the maximum possible rate, its motion during the first 20 s is extremely well modeled by the simple equation

$$v_x^2 = \frac{2P}{m} t$$

where $P = 3.6 \times 10^4$ watts is the car's power output, $m = 1200$ kg is its mass, and v_x is in m/s. That is, the square of the car's velocity increases linearly with time.
    a. Find an algebraic expression in terms of P, m, and t for the car's acceleration at time t.
    b. What is the car's speed at $t = 2$ s and $t = 10$ s?
    c. Evaluate the acceleration at $t = 2$ s and $t = 10$ s.
75. || The two masses in FIGURE P2.75 slide on frictionless wires. They are connected by a pivoting rigid rod of length L. Prove that $v_{2x} = -v_{1y} \tan\theta$.

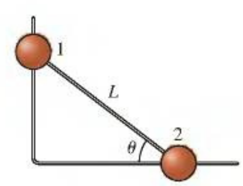


FIGURE P2.75

In Problems 76 through 79, you are given the kinematic equation or equations that are used to solve a problem. For each of these, you are to:
  a. Write a *realistic* problem for which this is the correct equation(s). Be sure that the answer your problem requests is consistent with the equation(s) given.
  b. Draw the pictorial representation for your problem.
  c. Finish the solution of the problem.

76. $64\text{ m} = 0\text{ m} + (32\text{ m/s})(4\text{ s} - 0\text{ s}) + \frac{1}{2}a_x(4\text{ s} - 0\text{ s})^2$
77. $(10\text{ m/s})^2 = v_{0y}^2 - 2(9.8\text{ m/s}^2)(10\text{ m} - 0\text{ m})$
78. $(0\text{ m/s})^2 = (5\text{ m/s})^2 - 2(9.8\text{ m/s}^2)(\sin 10°)(x_1 - 0\text{ m})$
79. $v_{1x} = 0\text{ m/s} + (20\text{ m/s}^2)(5\text{ s} - 0\text{ s})$
    $x_1 = 0\text{ m} + (0\text{ m/s})(5\text{ s} - 0\text{ s}) + \frac{1}{2}(20\text{ m/s}^2)(5\text{ s} - 0\text{ s})^2$
    $x_2 = x_1 + v_{1x}(10\text{ s} - 5\text{ s})$

Challenge Problems

80. ||| A rocket is launched straight up with constant acceleration. Four seconds after liftoff, a bolt falls off the side of the rocket. The bolt hits the ground 6.0 s later. What was the rocket's acceleration?
81. ||| Careful measurements have been made of Olympic sprinters in the 100 meter dash. A simple but reasonably accurate model is that a sprinter accelerates at 3.6 m/s^2 for $3\frac{1}{3}$ s, then runs at constant velocity to the finish line.
    a. What is the race time for a sprinter who follows this model?
    b. A sprinter could run a faster race by accelerating faster at the beginning, thus reaching top speed sooner. If a sprinter's top speed is the same as in part a, what acceleration would he need to run the 100 meter dash in 9.9 s?
    c. By what percent did the sprinter need to increase his acceleration in order to decrease his time by 1%?

82. ||| Careful measurements have been made of Olympic sprinters
CALC in the 100 meter dash. A quite realistic model is that the sprinter's velocity is given by

$$v_x = a(1 - e^{-bt})$$

where t is in s, v_x is in m/s, and the constants a and b are characteristic of the sprinter. Sprinter Carl Lewis's run at the 1987 World Championships is modeled with $a = 11.81$ m/s and $b = 0.6887\ \text{s}^{-1}$.

a. What was Lewis's acceleration at $t = 0$ s, 2.00 s, and 4.00 s?
b. Find an expression for the distance traveled at time t.
c. Your expression from part b is a transcendental equation, meaning that you can't solve it for t. However, it's not hard to use trial and error to find the time needed to travel a specific distance. To the nearest 0.01 s, find the time Lewis needed to sprint 100.0 m. His official time was 0.01 s more than your answer, showing that this model is very good, but not perfect.

83. ||| A sprinter can accelerate with constant acceleration for 4.0 s before reaching top speed. He can run the 100 meter dash in 10.0 s. What is his speed as he crosses the finish line?

84. ||| A rubber ball is shot straight up from the ground with speed v_0. Simultaneously, a second rubber ball at height h directly above the first ball is dropped from rest.

a. At what height above the ground do the balls collide? Your answer will be an *algebraic expression* in terms of h, v_0, and g.
b. What is the maximum value of h for which a collision occurs before the first ball falls back to the ground?
c. For what value of h does the collision occur at the instant when the first ball is at its highest point?

85. ||| The Starship Enterprise returns from warp drive to ordinary space with a forward speed of 50 km/s. To the crew's great surprise, a Klingon ship is 100 km directly ahead, traveling in the same direction at a mere 20 km/s. Without evasive action, the Enterprise will overtake and collide with the Klingons in just slightly over 3.0 s. The Enterprise's computers react instantly to brake the ship. What magnitude acceleration does the Enterprise need to just barely avoid a collision with the Klingon ship? Assume the acceleration is constant.

Hint: Draw a position-versus-time graph showing the motions of both the Enterprise and the Klingon ship. Let $x_0 = 0$ km be the location of the Enterprise as it returns from warp drive. How do you show graphically the situation in which the collision is "barely avoided"? Once you decide what it looks like graphically, express that situation mathematically.